AD-A065 978 AIR FORCE WEAPONS LAB KIRTLAND AFB N MEX F/G 20/5
GAUSSIAN JITTER OF A FOCUSED BEAM OF LIGHT.(U)
APR 76 W T WHITE, J P BAUMGARDNER, D A HOLMES

UNCLASSIFIED AFWL-TR-76-66 SBIE-AD-E200 250 NL

1 OF 2
ADA
065978

1 OF 2

ADA 065978

1·0 2·8 2·5 3·2 2·2 3·6 2·0 4·0 1·1 1·8 1·25 1·4 1·6

NATIONAL BUREAU OF STANDARDS
MICROCOPY RESOLUTION TEST CHART

AD-E200 250

AD-A065978

AFWL-TR-76-66

[redacted]WL-TR-76-66 c.1

1

LEVEL III

GAUSSIAN JITTER OF A FOCUSED BEAM OF LIGHT

LEVEL III

Warren T. White
John R. Baumgardner
Dale A. Holmes

April 1976

Final Report

Approved for public release; distribution unlimited.

AIR FORCE WEAPONS LABORATORY
Air Force Systems Command
Kirtland Air Force Base, NM 87117

79 02 16 119

The research was prepared by the Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico, under Job Order 317J1897. Capt. Warren T. White, III (ALO) was the Laboratory Project Officer-in-Charge.

When US Government drawings, specifications, or other data are used for any purpose other than a definitely related Governnment procurement operation, the Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This technical note has been reviewed and is approved.

Warren T. White, III

WARREN T. WHITE, III
Capt, USAF
Project Officer

Armand D. Maio

ARMAND D. MAIO
Lt Col, USAF
Chief, Optics Branch
x8193

FOR THE COMMANDER

Robert D. Rose

ROBERT D. ROSE
Colonel, USAF
Chief, Advanced Laser Technology Division

DISCLAIMER NOTICE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFWL-TR-76-66	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) GAUSSIAN JITTER OF A FOCUSED BEAM OF LIGHT		5. TYPE OF REPORT & PERIOD COVERED Final Report
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Warren T. White John P. Baumgardner Dale A. Holmes		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Air Force Weapons Laboratory (ALO) Kirtland Air Force Base, NM 87117		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 63000F, 317J/18/97
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Weapons Laboratory Kirtland Air Force Base, NM 87117		12. REPORT DATE April 1976
		13. NUMBER OF PAGES 104
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Beam Wander
Laser Beam Wander
Jitter
Beam Jitter
Laser Beam Jitter
Laser Far-Field
Gaussian Jitter

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Random beam wander of the Fraunhoffer pattern of an annular beam of light of uniform irradiance and phase is modeled by convolving that pattern and a circularly symmetric gaussian probability density function. By substituting the function $\exp(\xi\rho^2/\sigma^2)/(\pi\sigma^2)$ for $2J_1(\pi\rho)/(\pi\rho)$ an expression is obtained which, within 5 percent, approximates the central lobe of the effective jittered irradiance distribution. Graphs are presented of effective irradiance profile, of jittered encircled power as a function of circle radius, and of (over)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

effective peak irradiance as a function of jitter. Annuli with area obscuration ratios of 0.0, 0.1, 0.2, and 0.3 are considered.

CONTENTS

ILLUSTRATIONS

ILLUSTRATIONS
(Cont.)

SECTION I

INTRODUCTION

Negro (1), Esposito (2), and others have proposed expressions to describe effects of jitter upon propagating, fundamental-mode, gaussian laser beams. No one has yet considered the effect of jitter on annular laser beams.

In many unstable resonator lasers, the output of the laser is approximately a beam of uniform phase and gaussian irradiance truncated by an annular aperture. As a first approximation to such a beam, this article considers a monochromatic beam of uniform phase and irradiance apertured by an annular slit.

Some effects of two-dimensional, random gaussian jitter on the Fraunhoffer pattern are examined. By two-dimensional, random gaussian jitter we mean a process which causes the center of the Fraunhoffer pattern to wander in such a way that the time-averaged probability density function for finding the pattern centered at any given location in the Fraunhoffer plane is gaussian. The main effect of jitter is to smear the irradiance distribution. The time-averaged Fraunhoffer pattern loses its characteristic fringes, the time-averaged peak irradiance diminishes, and in general the amount of power transmitted by a circular hole that is concentric with the center of the time-averaged Fraunhoffer irradiance pattern diminishes as jitter increases in amplitude.

Section II of this article presents a simple jitter model. Section III looks at the effect of jitter upon time-averaged power transmitted by a

circular aperture centered in the Fraunhoffer plane. Section IV looks at the effect of jitter upon peak irradiance of the time-averaged Fraunhoffer pattern. Section V presents graphs of the effective, time-averaged Fraunhoffer irradiance distribution, <I>, as a function of radial coordinate and of jitter. It concludes the paper by presenting an approximation to <I>.

SECTION II

THE MODEL

The fundamental equation of this paper is the following two-dimensional convolution of the circularly symmetric functions $I(\rho;\alpha)$ and $p(\rho;\sigma)$:

$$\langle I\rangle = I(\rho;\alpha) \star p(\rho;\sigma)$$
$$= \int_0^\infty \int_0^{2\pi} I\left(\sqrt{\rho^2 + \rho_o^2 - 2\,\rho\rho_o\cos\theta_o};\alpha\right) p(\rho_o;\sigma)\rho_o d\theta_o d\rho_o \qquad (1)$$

where

$$I(\rho;\alpha) = \frac{4I_o}{(1-\alpha^2)^2} \left\{ \frac{J_1(\pi\rho) - \alpha J_1(\pi\alpha\rho)}{\pi\rho} \right\}^2 \qquad (2)$$

and

$$p(\rho;\sigma) = \frac{\exp(-\rho^2/\sigma^2)}{\pi\sigma^2} \qquad (3)$$

$I(\rho;\alpha)$ is the unjittered Fraunhoffer irradiance pattern caused by focusing a plane wave of uniform irradiance through an annular aperture having a ratio of inner diameter to outer diameter of α (figure 1). We shall refer to α^2 as the obscuration ratio of the aperture.

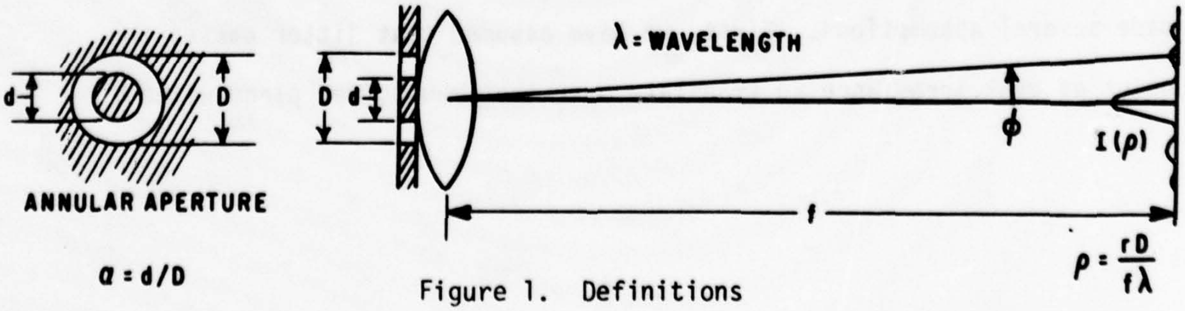


Figure 1. Definitions

I_o is the peak Fraunhoffer irradiance. ρ is a dimensionless far-field coordinate given by

$$\rho = \frac{rD}{f\lambda} \tag{4}$$

where D is the outer diameter of the aperture, f is the distance from the aperture to the Fraunhoffer plane, λ is the wavelength of radiation, and r is the distance from the origin of the Fraunhoffer plane to any arbitrary point in the Fraunhoffer plane. The function $p(\rho;\sigma)$ is the time-averaged probability density function that the location of the peak value of the jittering irradiance distribution is inside of differential area $\rho\, d\theta\, d\rho$ which is located at (ρ,θ). In this article the parameter σ is defined as the rms jitter. It is dimensionless. Mathematically it is the square root of σ^2, where

$$\begin{aligned} \sigma^2 &= \langle \rho^2 \rangle \\ &= 2\pi \int_0^\infty \rho^2\ p(\rho,\sigma)\ \rho\ d\rho \end{aligned} \tag{5}$$

In the expression for $I(\rho;\alpha)$, we have assumed that we have a diffraction-limited focal spot and that $f \gg D \gg \lambda$ and $f \gg r$.

By approximating the effective irradiance, $\langle I \rangle$, as a convolution, we have made several assumptions. First, we have assumed that jitter causes the point of peak irradiance to translate over the Fraunhoffer plane without

distorting the irradiance distribution relative to that point. Secondly, we have assumed that the focal surface is truly a plane. Thirdly, we have assumed that infinitely large jitter displacements might occur. For a brief discussion of these last three assumptions, see Appendix A.

SECTION III
ENCIRCLED POWER AS A FUNCTION OF CIRCLE SIZE

Encircled power is the total radiant power contained within an illuminated circle. Let $\langle P(a)\rangle$ denote time-averaged encircled power over a circle of radius a that is centered at the origin of the Fraunhoffer plane.

$$\langle P(a)\rangle = 2\pi \left(\frac{f\lambda}{D}\right)^2 \int_0^{\frac{aD}{f\lambda}} \langle I\rangle \, \rho d\rho \tag{6}$$

The quantity $\langle P(a)\rangle / P_T$, where P_T is the total output power over the entire Fraunhoffer plane, is a measure of how well focused a beam of light is. The effect of jitter upon $\langle P(a)\rangle / P_T$ is generally to decrease its value. An exception occurs when jitter is slight and the rim of the circle is located near a fringe. In such cases, a small amount of jitter tends to increase the encircled power.

The graphs in figures 2(a)-2(d) depict encircled power as a function of circle size. The ordinates are normalized so that an infinitely large circle ($aD/f\lambda = \infty$) causes the function to equal unity. Each graph consists of a family of curves for a given obscuration ratio, α^2. Each curve in a family represents $\langle P(a)\rangle / P_T$ as a function of the dimensionless variable $aD/f\lambda$ for a fixed amount of rms jitter.

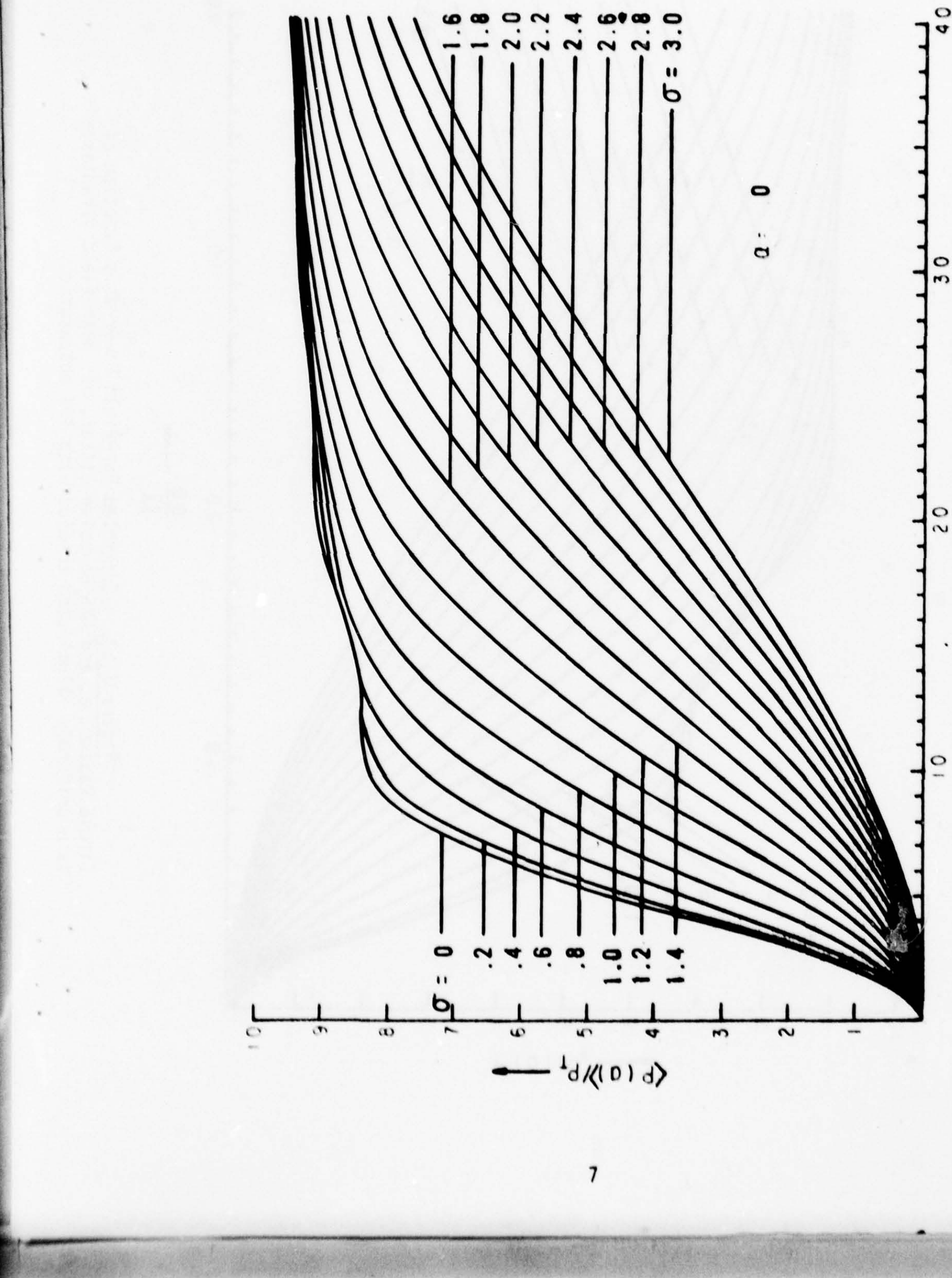


Figure 2(a). Encircled Far-Field Power as a Function of Circle Radius, a, and RMS Gaussian Jitter, σ. Near-field irradiance is a uniformly illuminated circle.

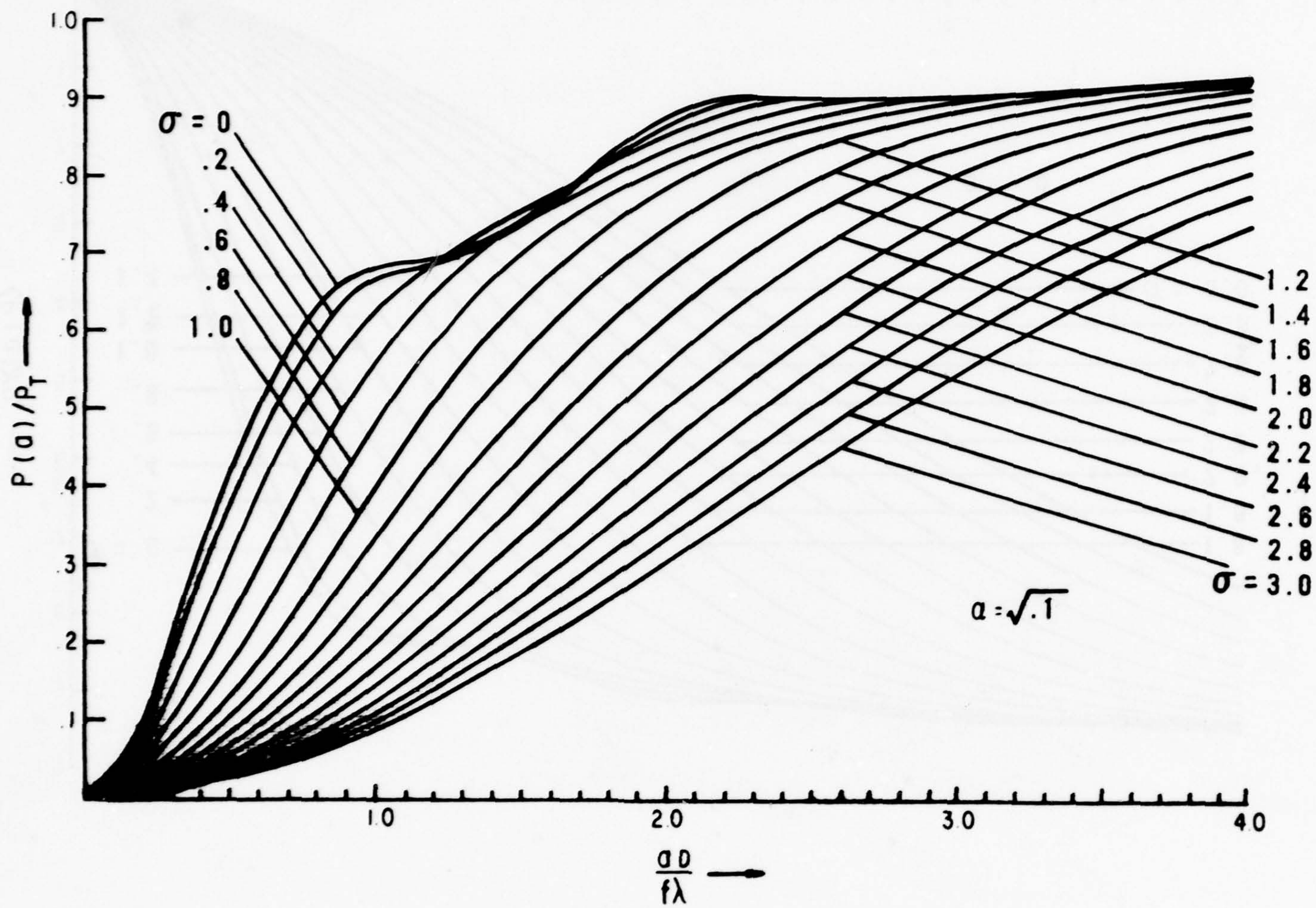


Figure 2(b). Encircled Far-Field Power as a Function of Circle Radius, a, and RMS Gaussian Jitter, σ. Near-field irradiance is a uniformly illuminated annulus with 10% obscuration.

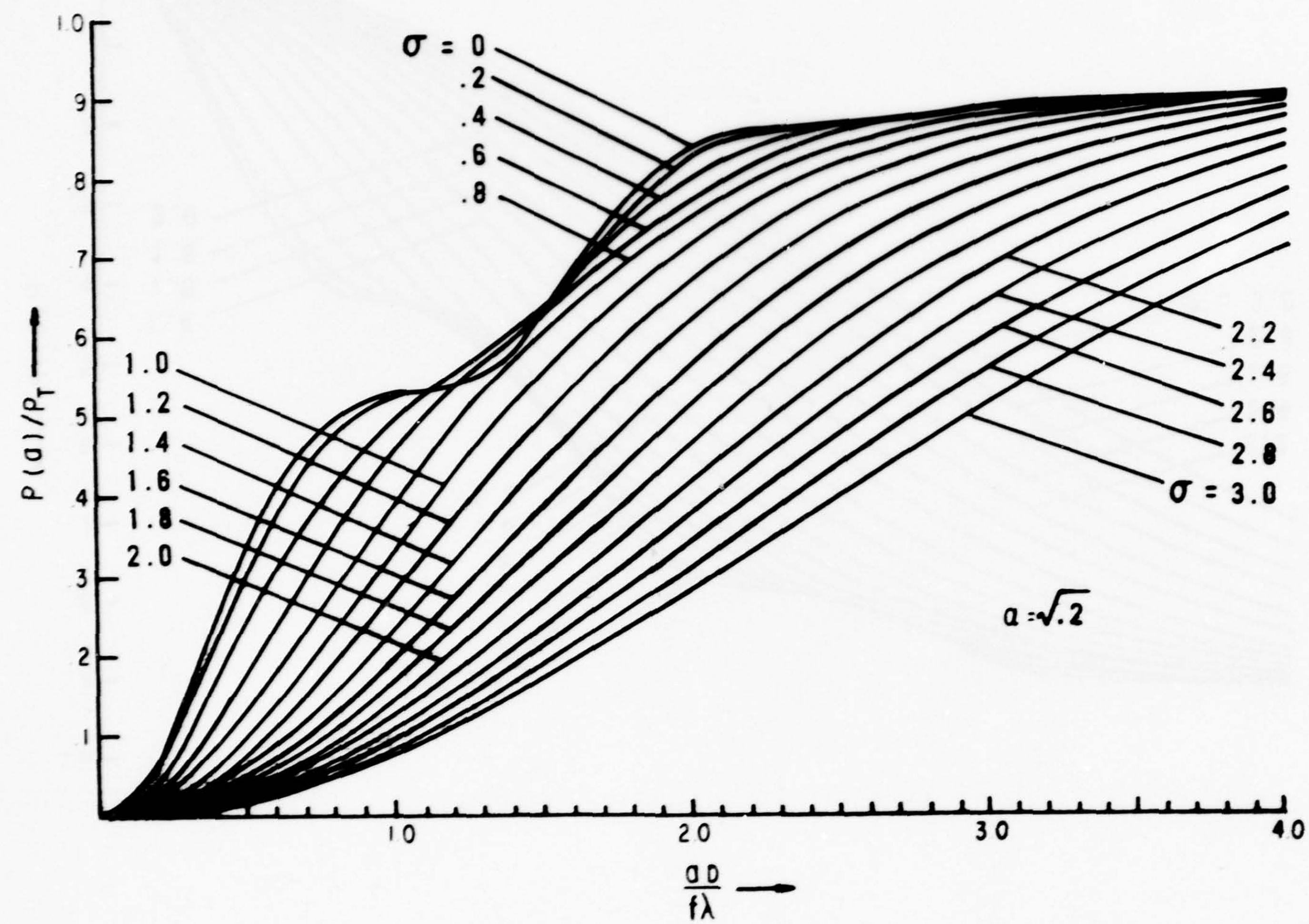


Figure 2(c). Encircled Far-Field Power as a Function of Circle Radius, a, and RMS Gaussian Jitter, σ. Near-field irradiance is a uniformly illuminated annulus with 20% obscuration.

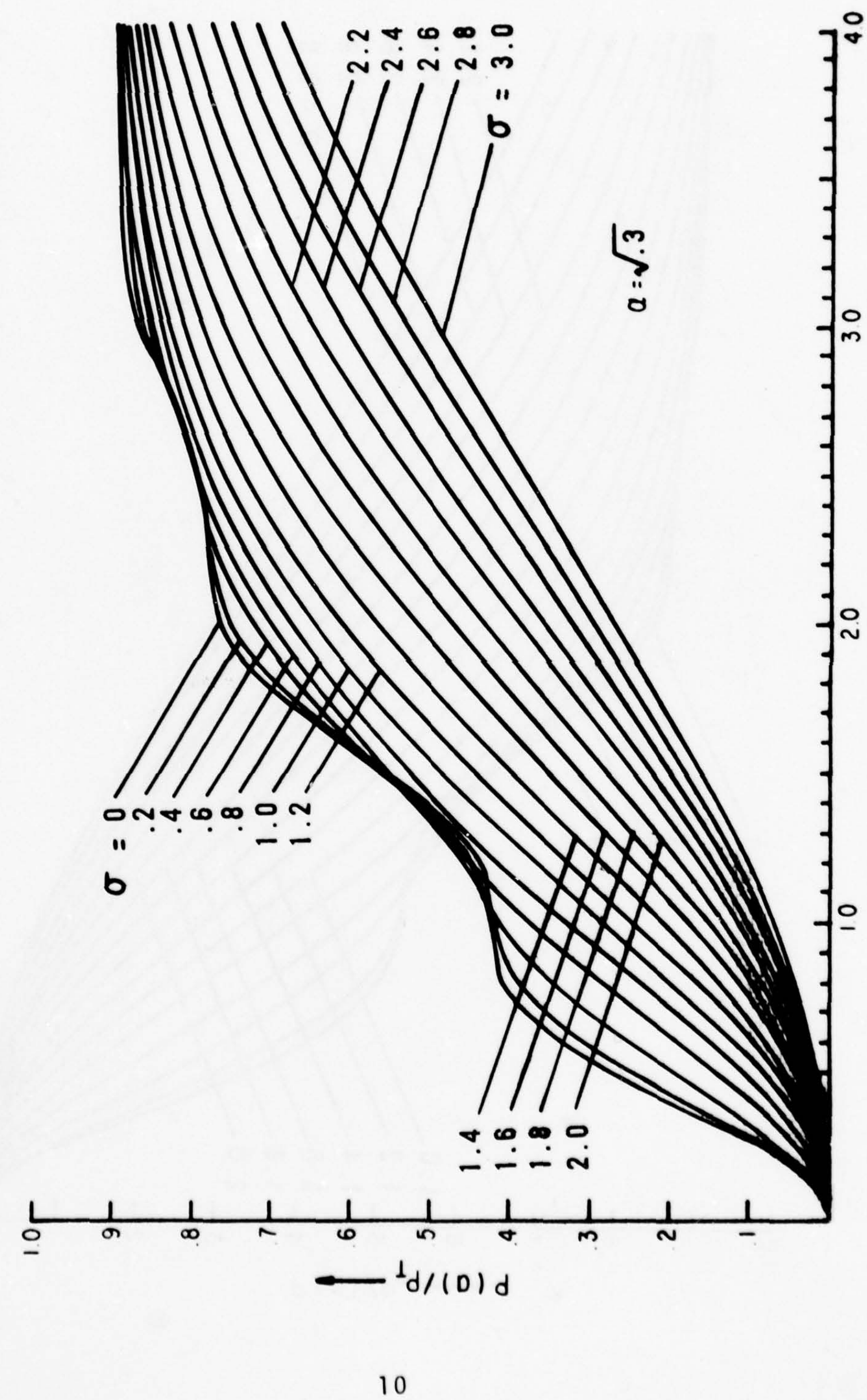


Figure 2(d). Encircled Far-Field Power as a Function of Circle Radius, a, and RMS Gaussian Jitter, σ. Near-field irradiance is a uniformly illuminated annulus with 30% obscuration.

Some general comments apply to all of these graphs. First, on each graph the curve corresponding to no jitter tends to have points of zero slope, corresponding to fringes. As jitter increases these points of zero slope disappear, corresponding to fringe wash-out. Secondly, given a value of $^{aD}/f\lambda$, an increase in rms jitter, σ, tends to decrease encircled power except as previously noted. However, given a large enough circle, the effect of jitter, for small enough jitter, is unnoticeable. For example, figure 2(a) shows that for any circle whose radius, a, is larger than $a' = 2f\lambda/D$, $\langle P(a) \rangle / P_T$ is independent of the rms jitter, σ, to within a tolerance of about two percent as long as $\sigma \lesssim 0.5$. For a larger value of a', say $a' = 3f\lambda/D$, $\langle P(a) \rangle / P_T$ is approximately independent of σ for all a such that $a \geq a'$ and for $\sigma \lesssim 1.0$. In less mathematical terms, since most of the power in the focused laser beam is near the point of peak irradiance, if the point of peak irradiance, although it may be moving from point to point, stays well within the rim of a circle whose radius equals at least a', then the amount of power transmitted by that circle is approximately the same as it would be if the laser beam were not jittering.

SECTION IV

TIME-AVERAGED AXIAL FRAUNHOFFER IRRADIANCE, $\langle I(0)\rangle$

1. $\langle I(0)\rangle$ AS A FUNCTION OF σ

Figure 3 shows how time-averaged axial Fraunhoffer irradiance deteriorates if jitter exists. These curves are hand-smoothed lines fit through a discrete sequence of digital computer evaluations of the integral

$$\frac{\langle I(0)\rangle}{I_o} = \frac{2\pi}{I_o}\int_0^\infty I(\rho)\, e^{-\rho^2/\sigma^2} \rho \, d\rho \qquad (7)$$

The curves are normalized to unity for no jitter. The normalizing factor I_o is the peak instantaneous Fraunhoffer irradiance, as in equation (2).

As an illustration of the jitter-induced degradation of axial Fraunhoffer irradiance, consider an unstable optical resonator with uniformly intense, collimated annular output which is subsequently focused at f = 2.5 km and has

λ = 10 μm

D = 10 cm

α^2 = .3 (70% geometric output coupling).

Assume the beam vibrates in a random gaussian fashion with an rms angular jitter of 60 μrad. This is equivalent to assuming σ = 0.6. From figure 3

$$\left.\frac{\langle I(0)\rangle}{I_o}\right|_{\substack{\sigma = .6 \\ \alpha^2 = .3}} = .42$$

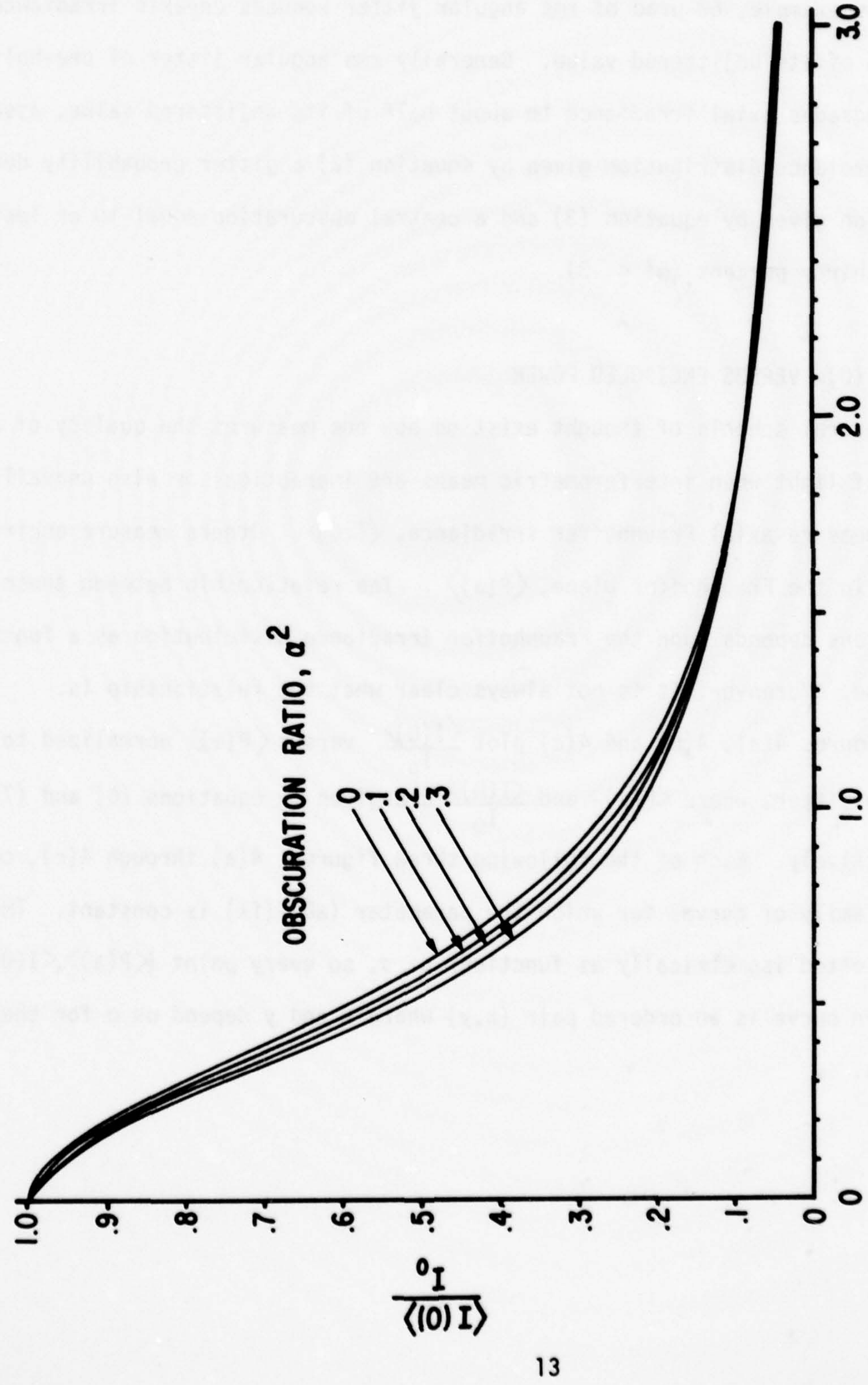


Figure 3. Degradation of Far-Field On-Axis Irradiance Due to an RMS Line-of-Sight Jitter, σ. RMS jitter is dimensionless but may be thought of as being in units of λ/D.

In this example, 60 μrad of rms angular jitter reduces on-axis irradiance to 42% of its unjittered value. Generally rms angular jitter of one-half λ/D degrades axial irradiance to about half of its unjittered value, assuming an irradiance distribution given by equation (2) a jitter probability density function given by equation (3) and a central obscuration equal to or less than thirty percent ($\alpha^2 \lesssim .3$).

2. $\langle I(0)\rangle$ VERSUS ENCIRCLED POWER

Several schools of thought exist on how one measures the quality of a beam of light when interferometric means are impractical or else unavailable. Some measure axial Fraunhoffer irradiance, $\langle I(0)\rangle$. Others measure encircled power in the Fraunhoffer plane, $\langle P(a)\rangle$. The relationship between these two functions depends upon the Fraunhoffer irradiance distribution as a function of time. Moreover, it is not always clear what the relationship is.

Figures 4(a), 4(b) and 4(c) plot $\frac{\langle I(0)\rangle}{I_o}$ versus $\langle P(a)\rangle$ normalized to unity for no jitter, where $\langle P(a)\rangle$ and $\frac{\langle I(0)\rangle}{I_o}$ are given by equations (6) and (7) respectively. Each of the following three figures, 4(a) through 4(c), consist of a family of curves for which the parameter $(aD)/(f\lambda)$ is constant. The data are plotted isometrically as functions of σ, so every point $(\langle P(a)\rangle, \langle I(0)\rangle)$ on a given curve is an ordered pair (x,y) where x and y depend on σ for their values.

The fact that all points corresponding to $\sigma \neq 0$ in following graphs fall below the line $y = x$ (where x equals $\langle P(a) \rangle$ normalized to unity for $\sigma = 0$ and where y equals $\langle I(0) \rangle$) shows that jitter degrades $\langle I(0) \rangle$ more than it does $\langle P(a) \rangle$.

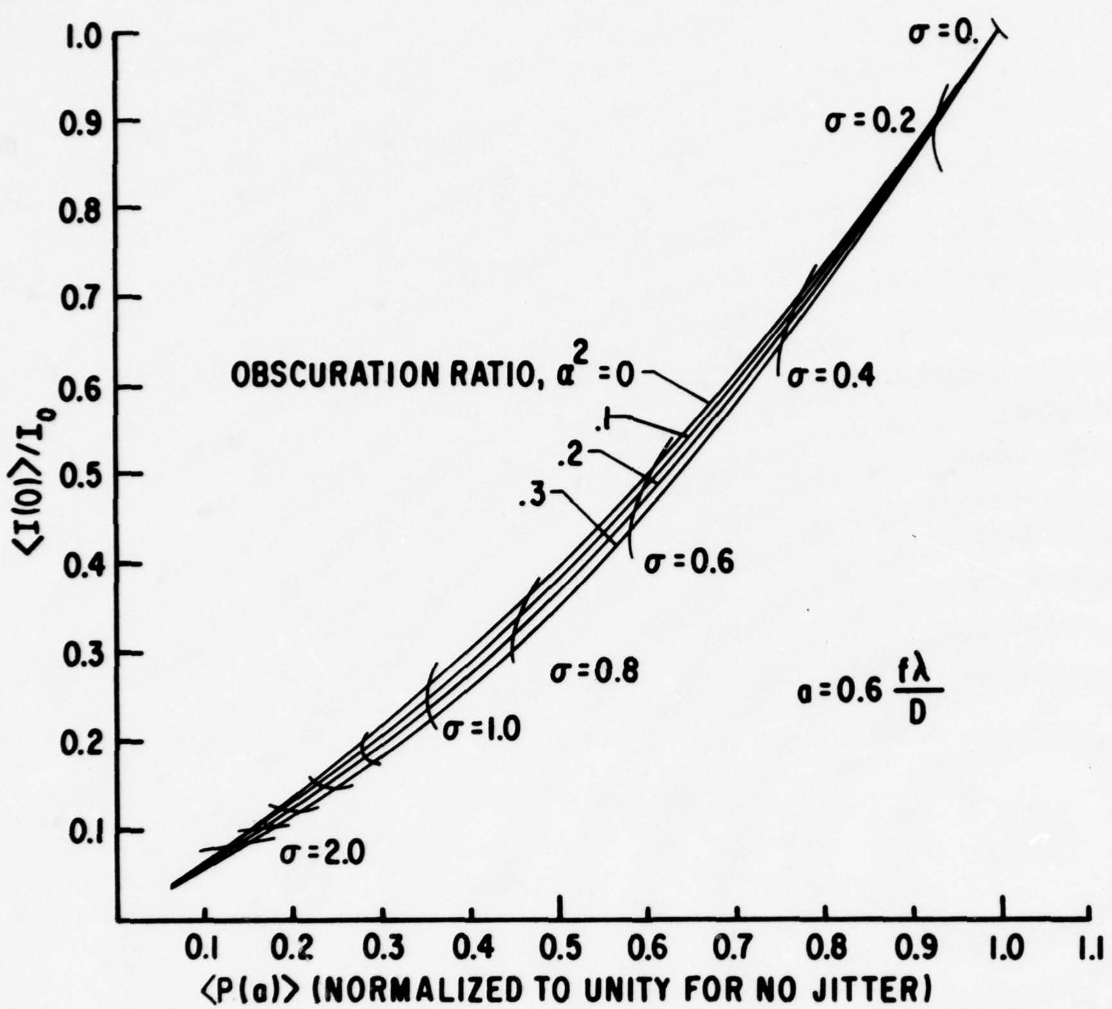


Figure 4(a). Degraded Far-Field On-Axis Irradiance Plotted Against Degraded Far-Field Encircled Power. The radius of the far-field circle is fixed at $aD/f\lambda = 0.6$.

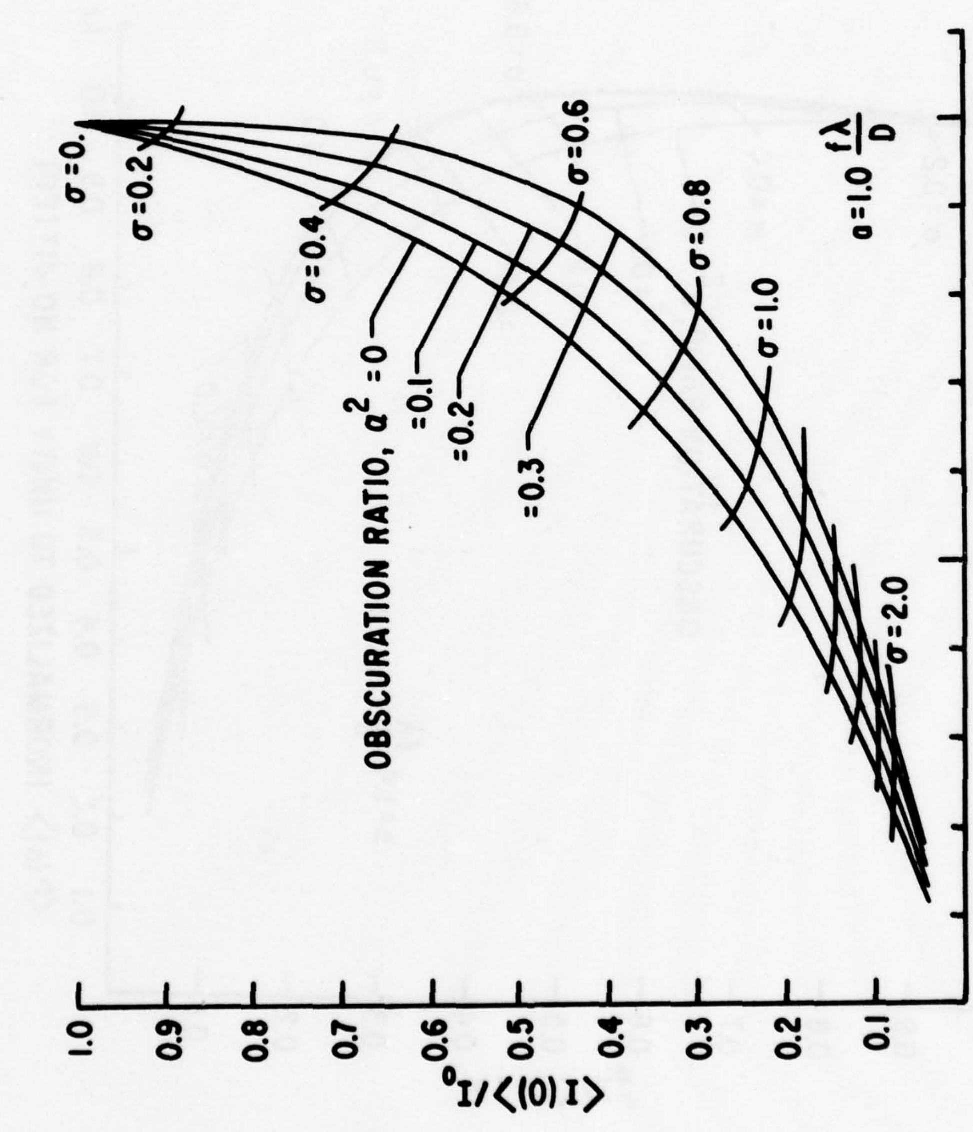


Figure 4(b). Degraded Far-Field On-Axis Irradiance Plotted Against Degraded Far-Field Encircled Power. The radius of the far-field circle is fixed at $aD/f\lambda = 1.0$.

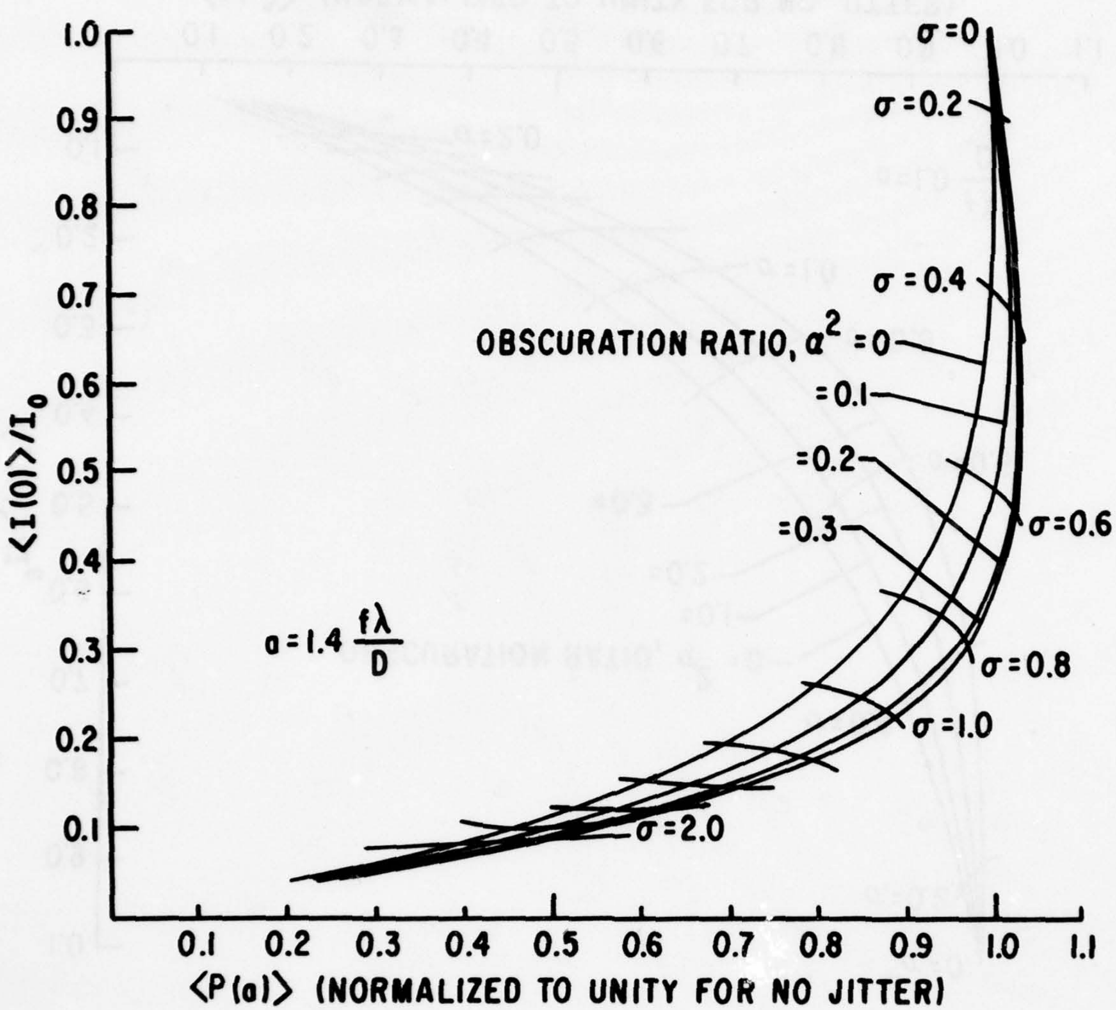


Figure 4(c). Degraded Far-Field On-Axis Irradiance Plotted Against Degraded Far-Field Encircled Power. The radius of the far-field circle is fixed at $aD/f\lambda = 1.4$.

SECTION V

TIME-AVERAGED FRAUNHOFFER PATTERN

1. $\langle I \rangle$ vs ρ

As rms jitter increases, the effective far-field irradiance distribution tends to smooth out. The first part of this section presents computer calculated graphs of the effective irradiance, $\langle I \rangle$, versus the normalized far-field coordinate, ρ.

The computer calculated values of $\langle I \rangle$ are plotted as a dashed line. Superimposed on each graph is a least-squares fit according to an approximation given in the last part of this section.

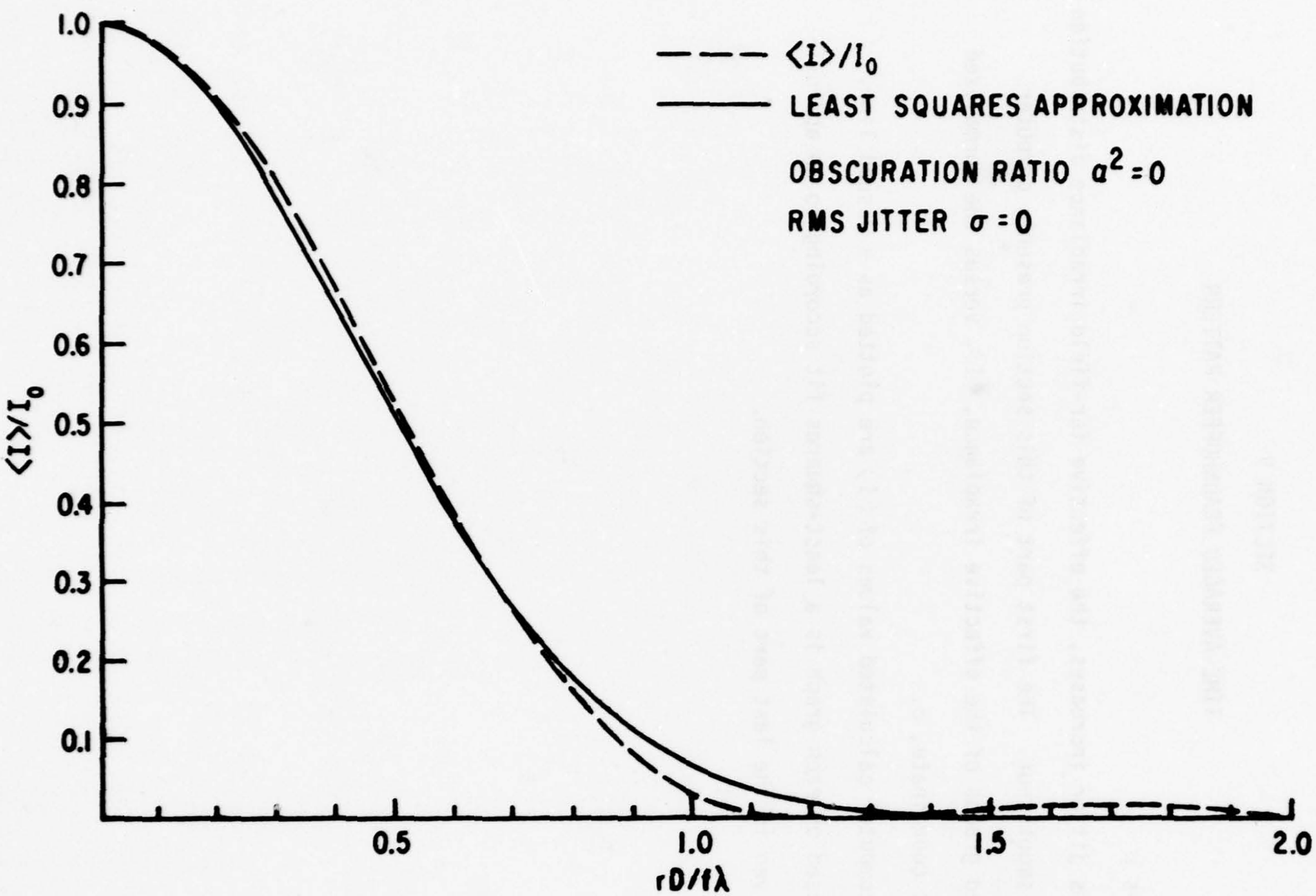


Figure 5(a)-1. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

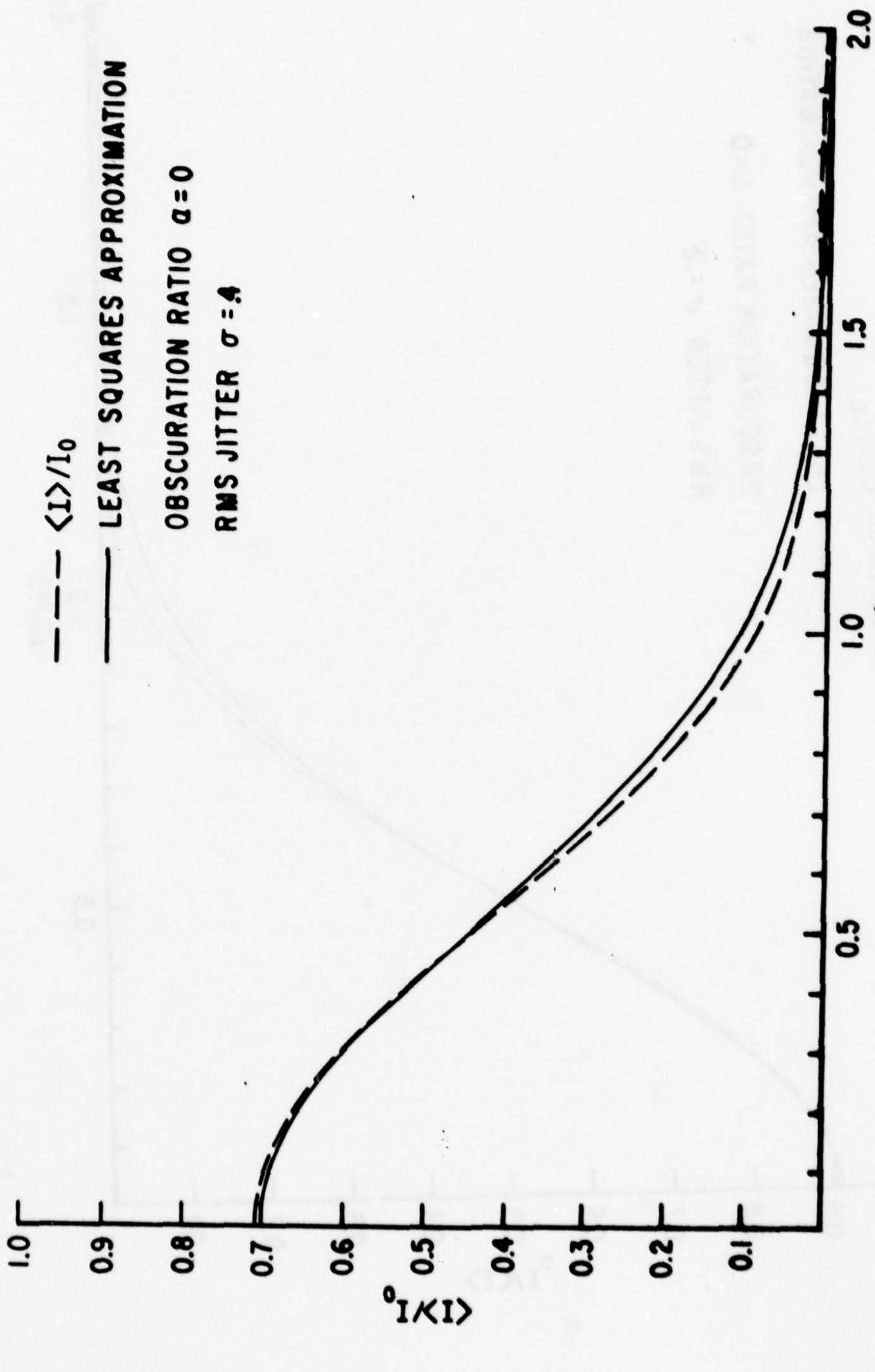


Figure 5(a)-2. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

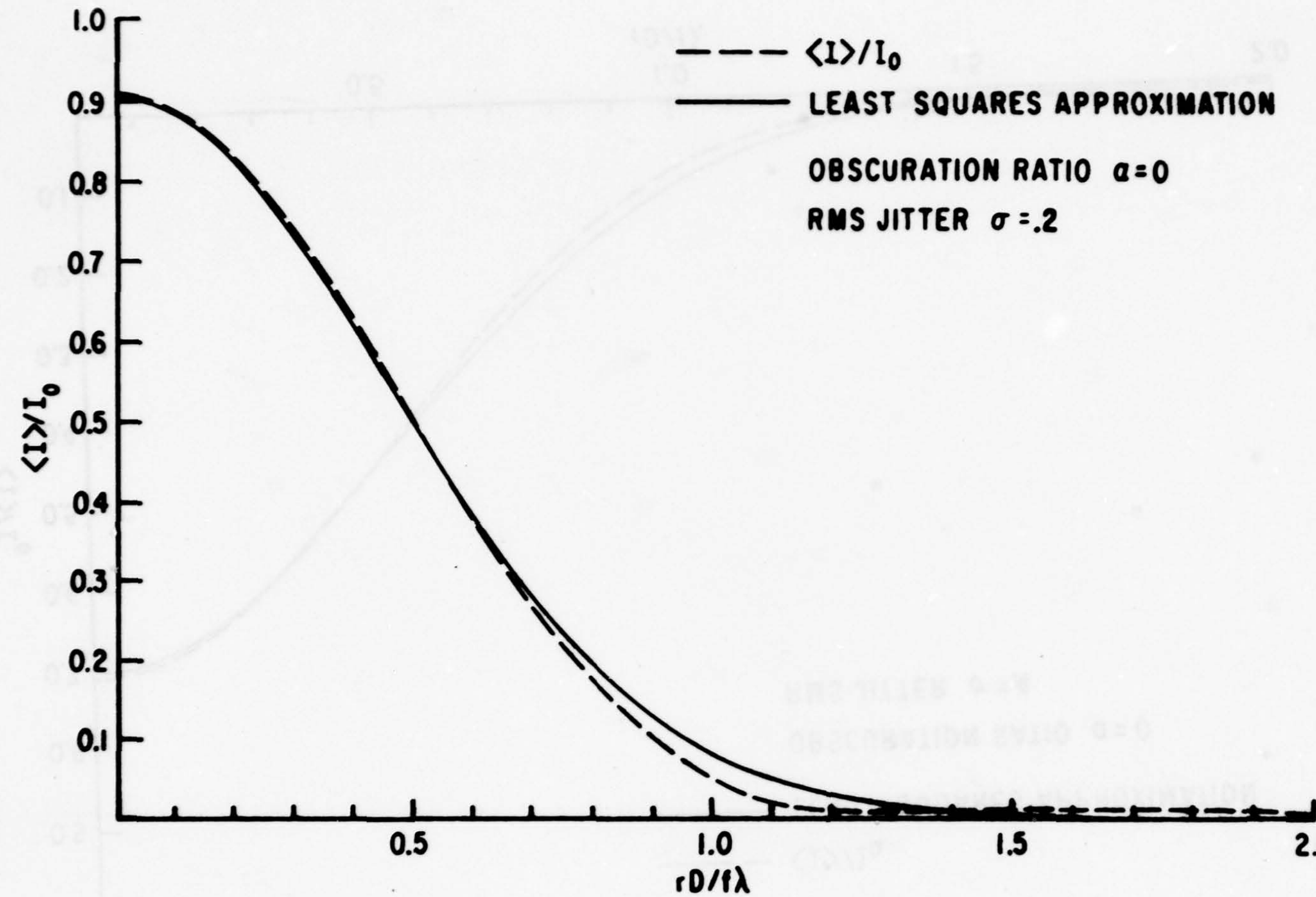


Figure 5(a)-3. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

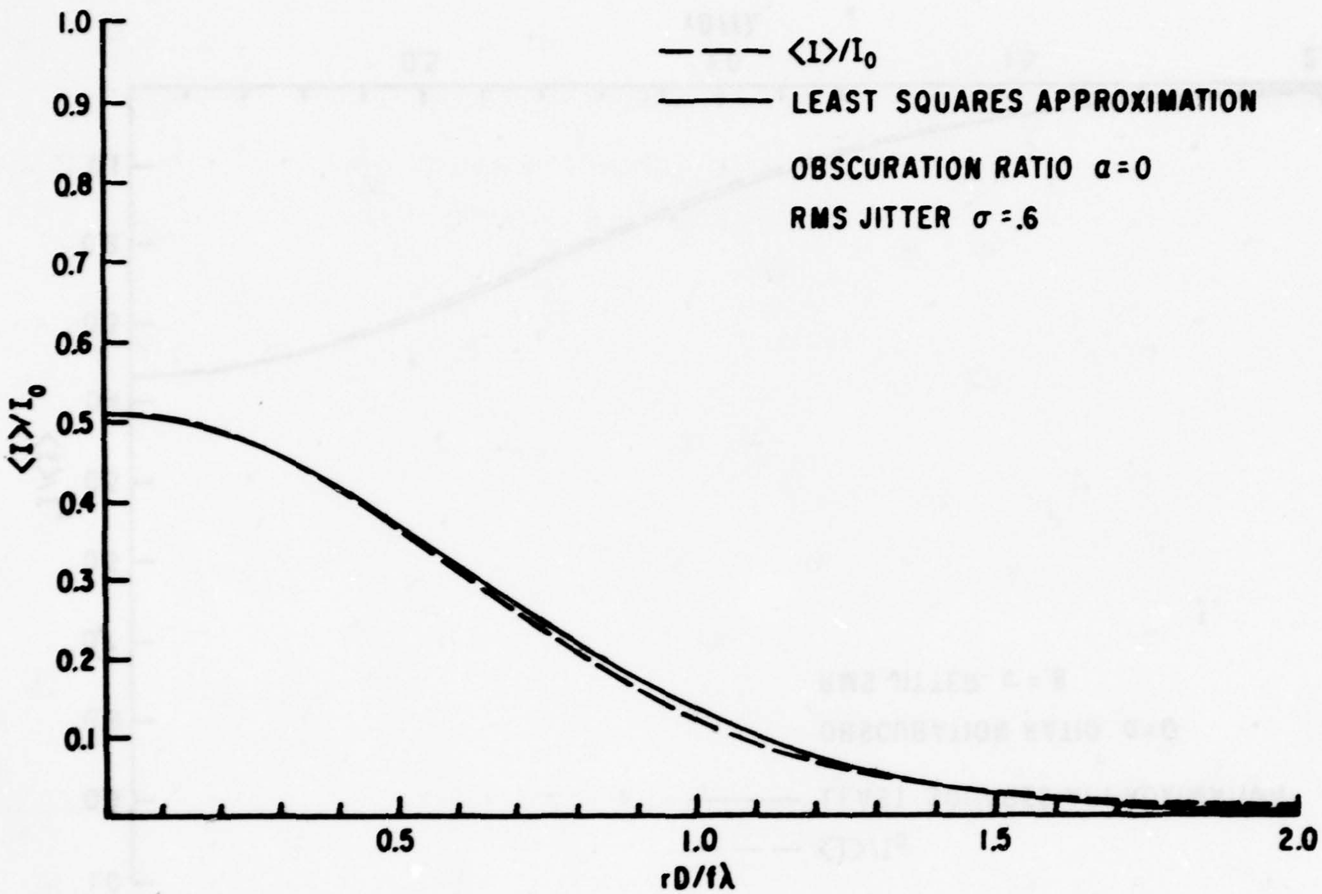


Figure 5(a)-4. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

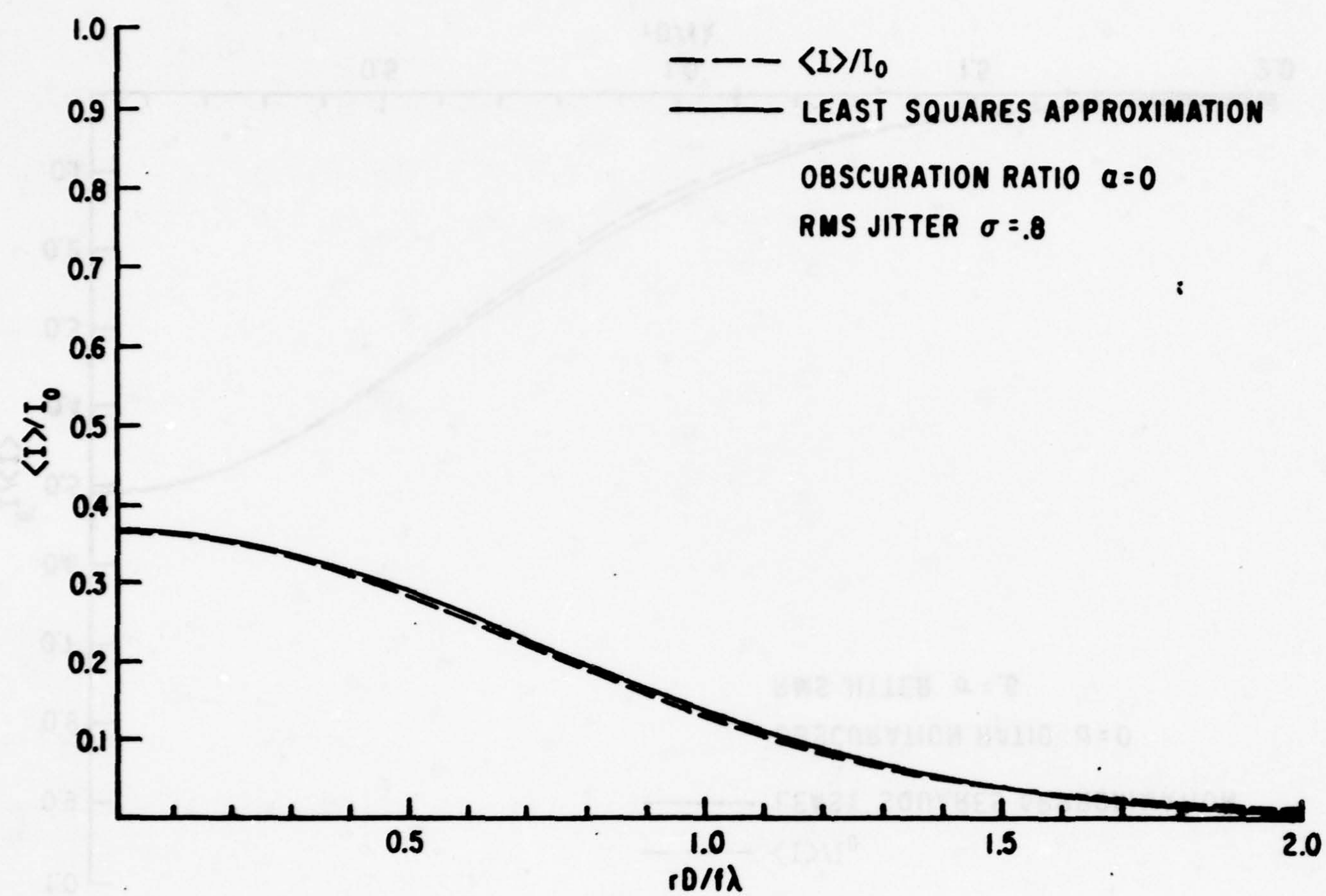


Figure 5(a)-5. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

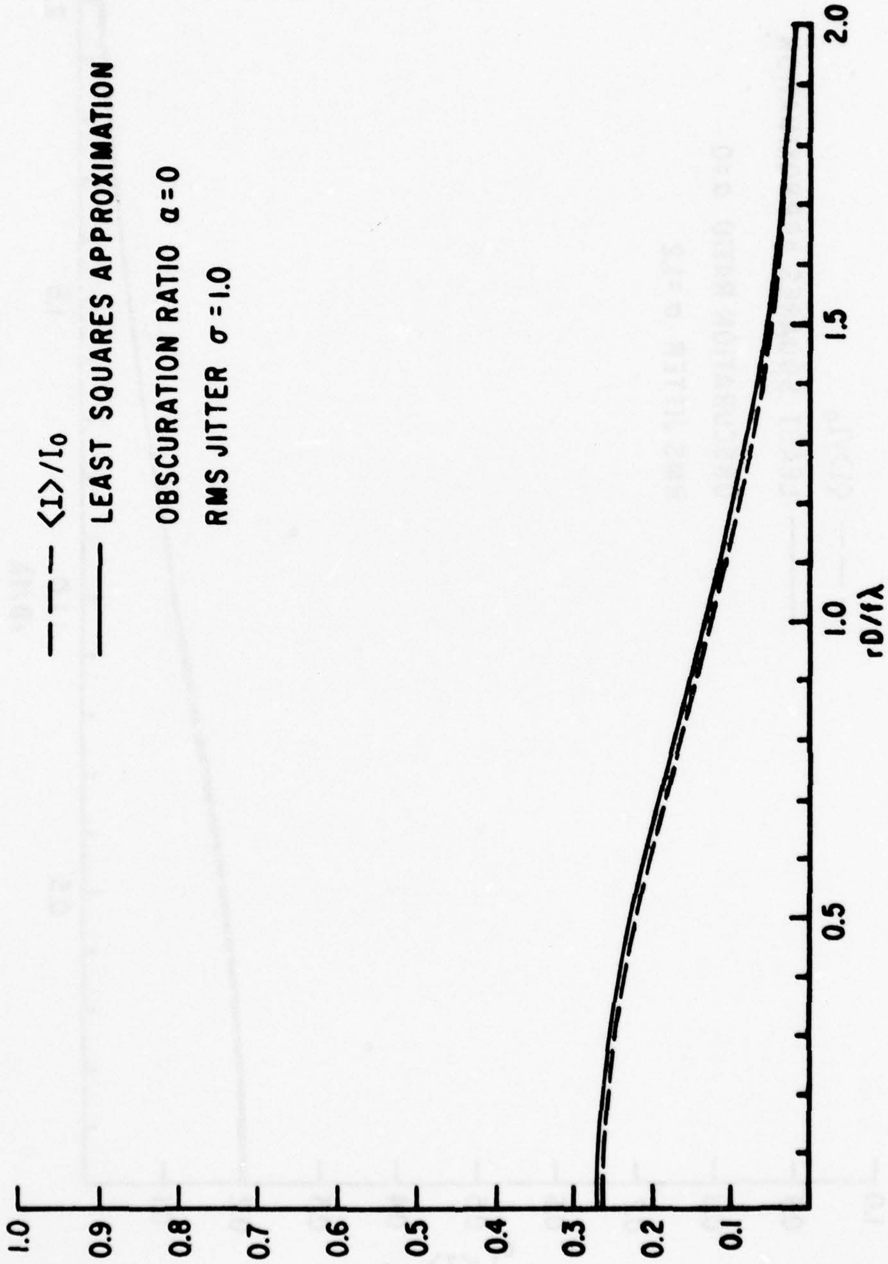


Figure 5(a)-6. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

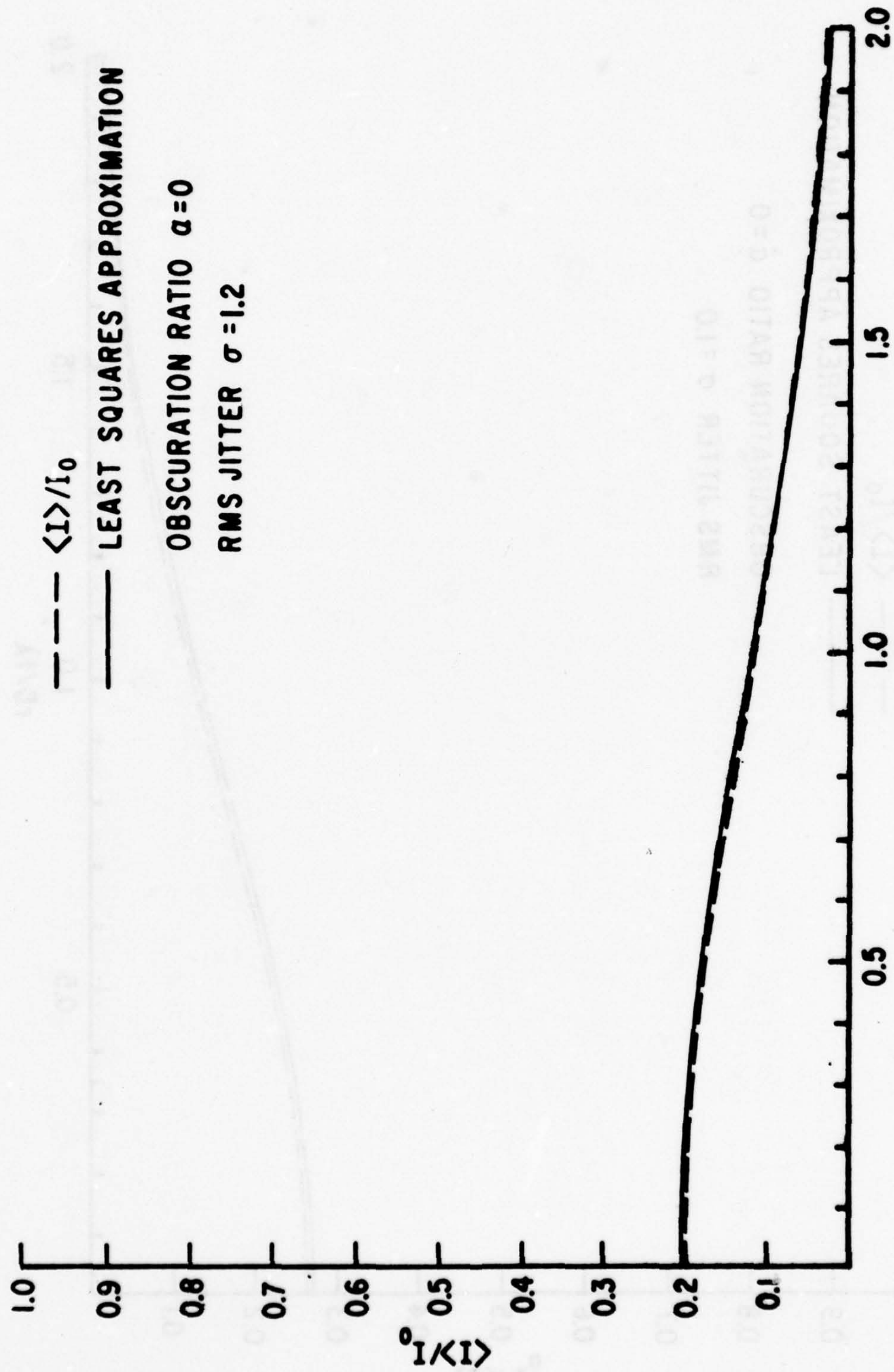


Figure 5(a)-7. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

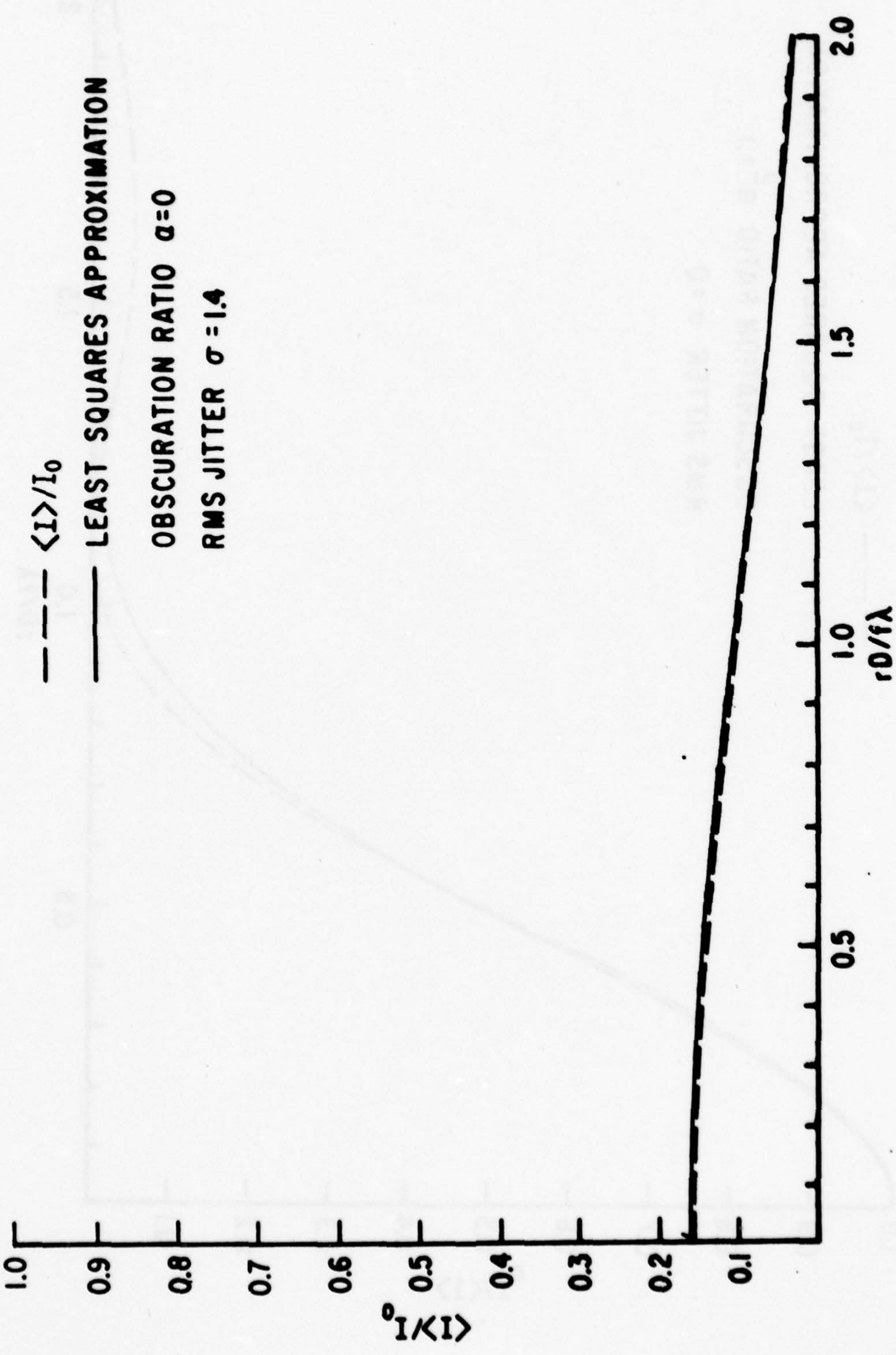


Figure 5(a)-8. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

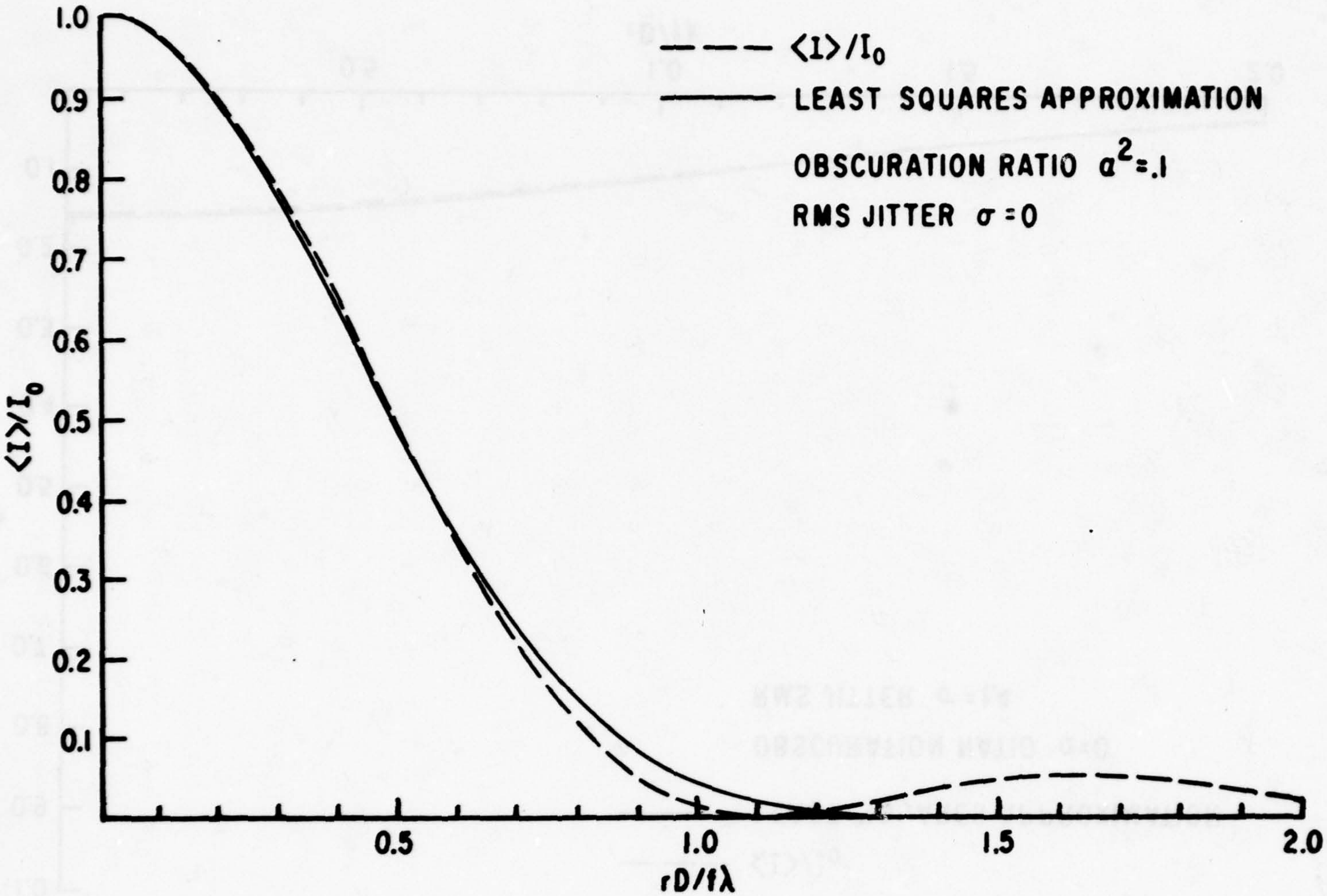


Figure 5(b)-1. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

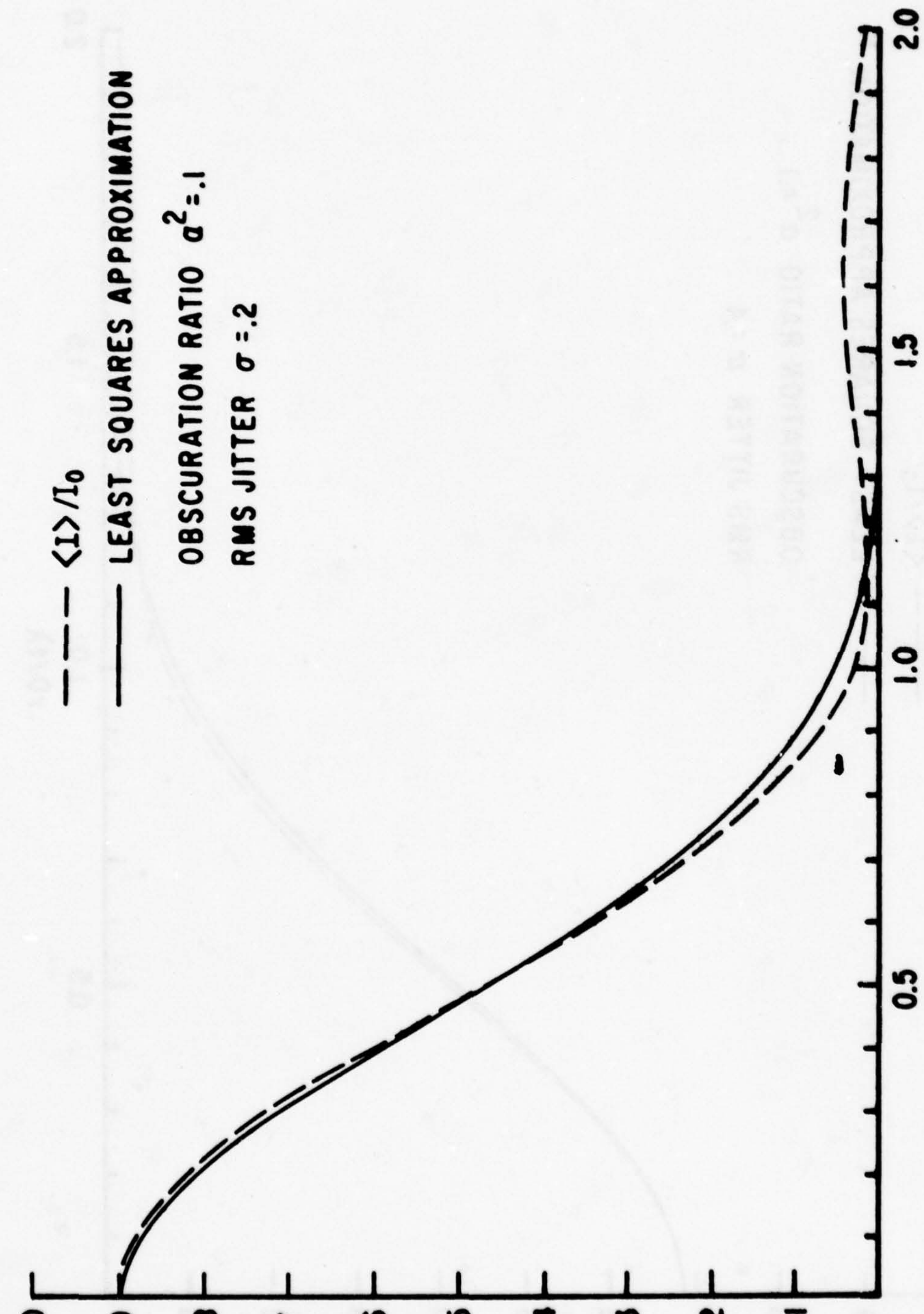


Figure 5(b)-2. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

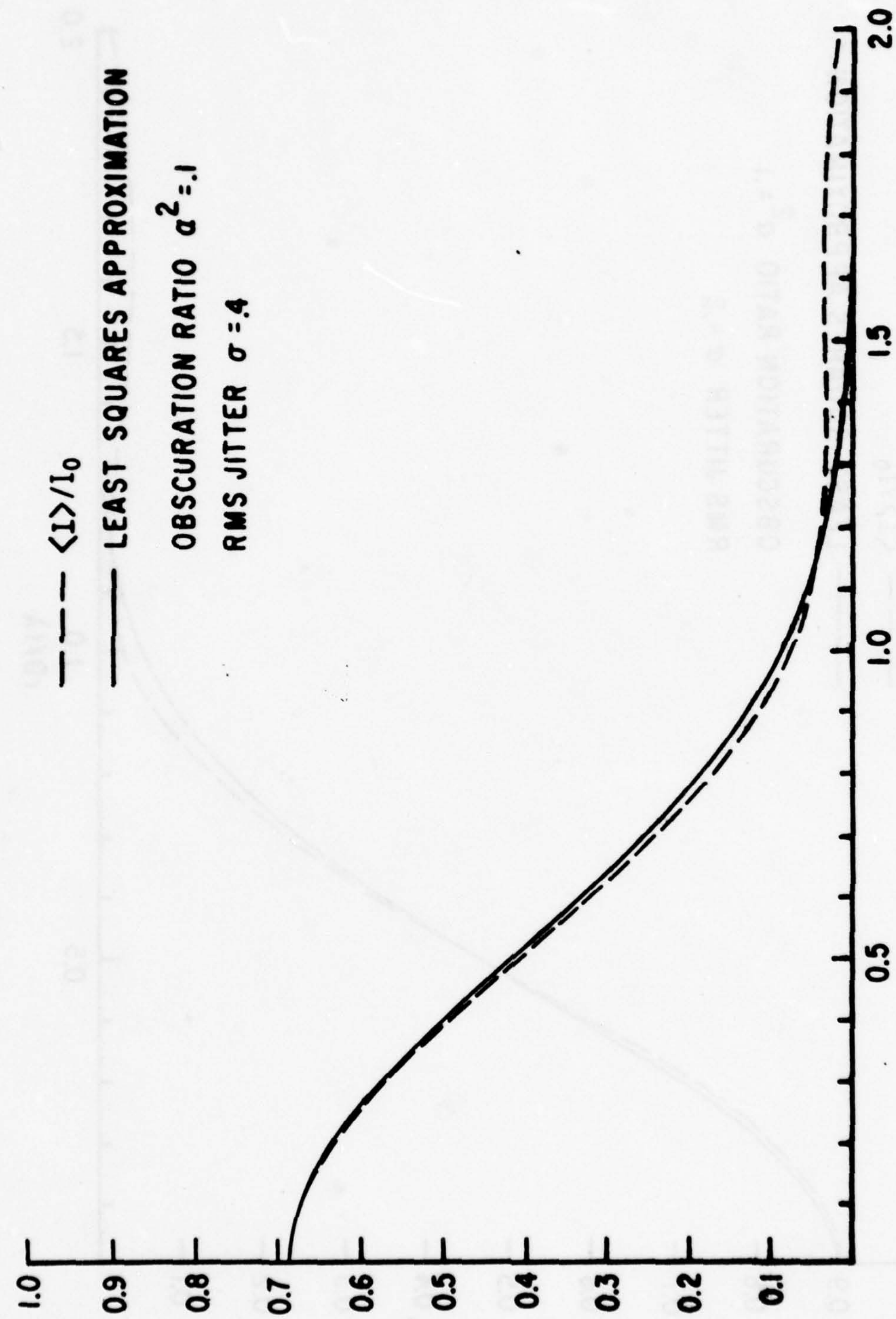


Figure 5(b)-3. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

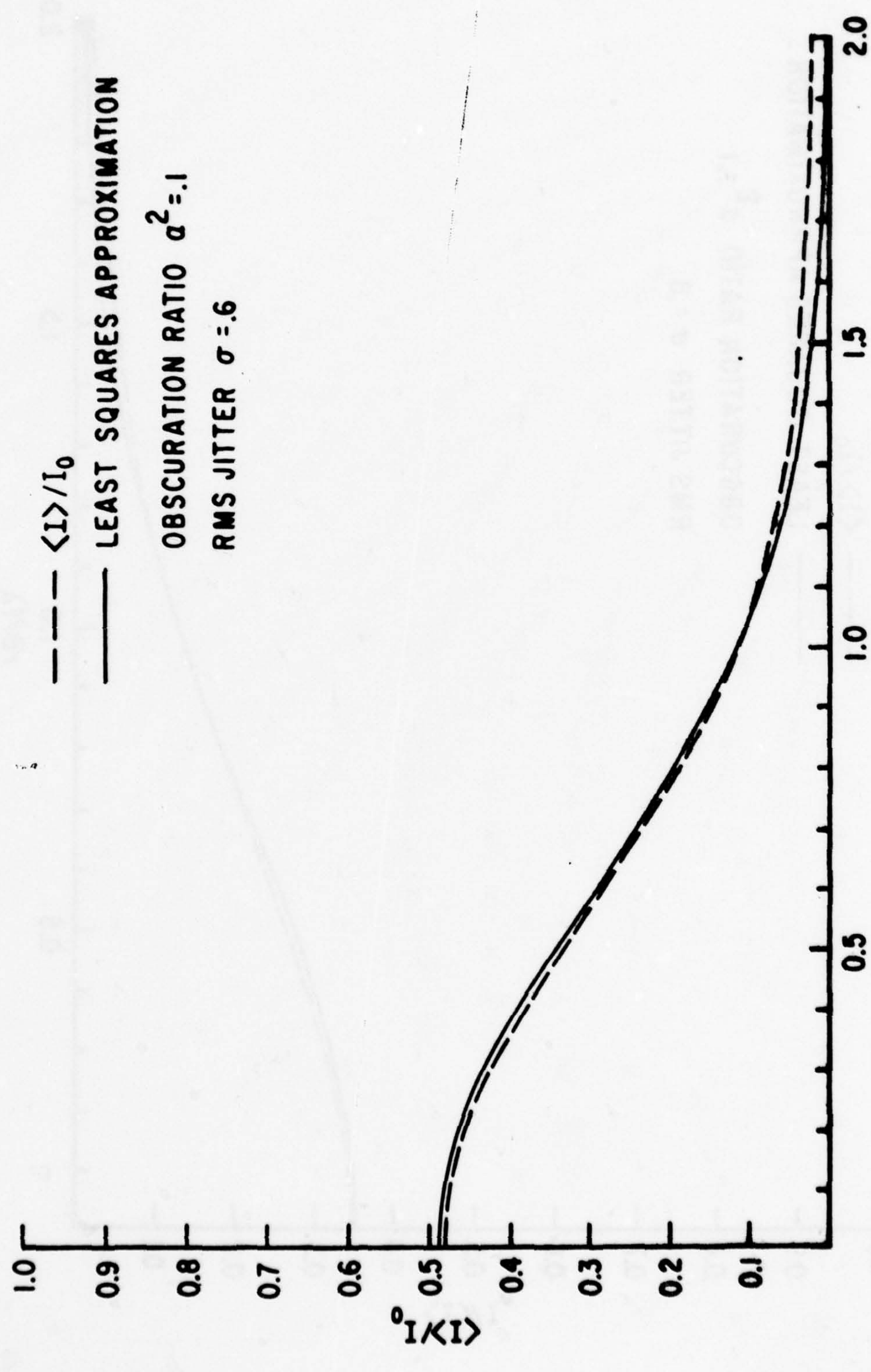


Figure 5(b)-4. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

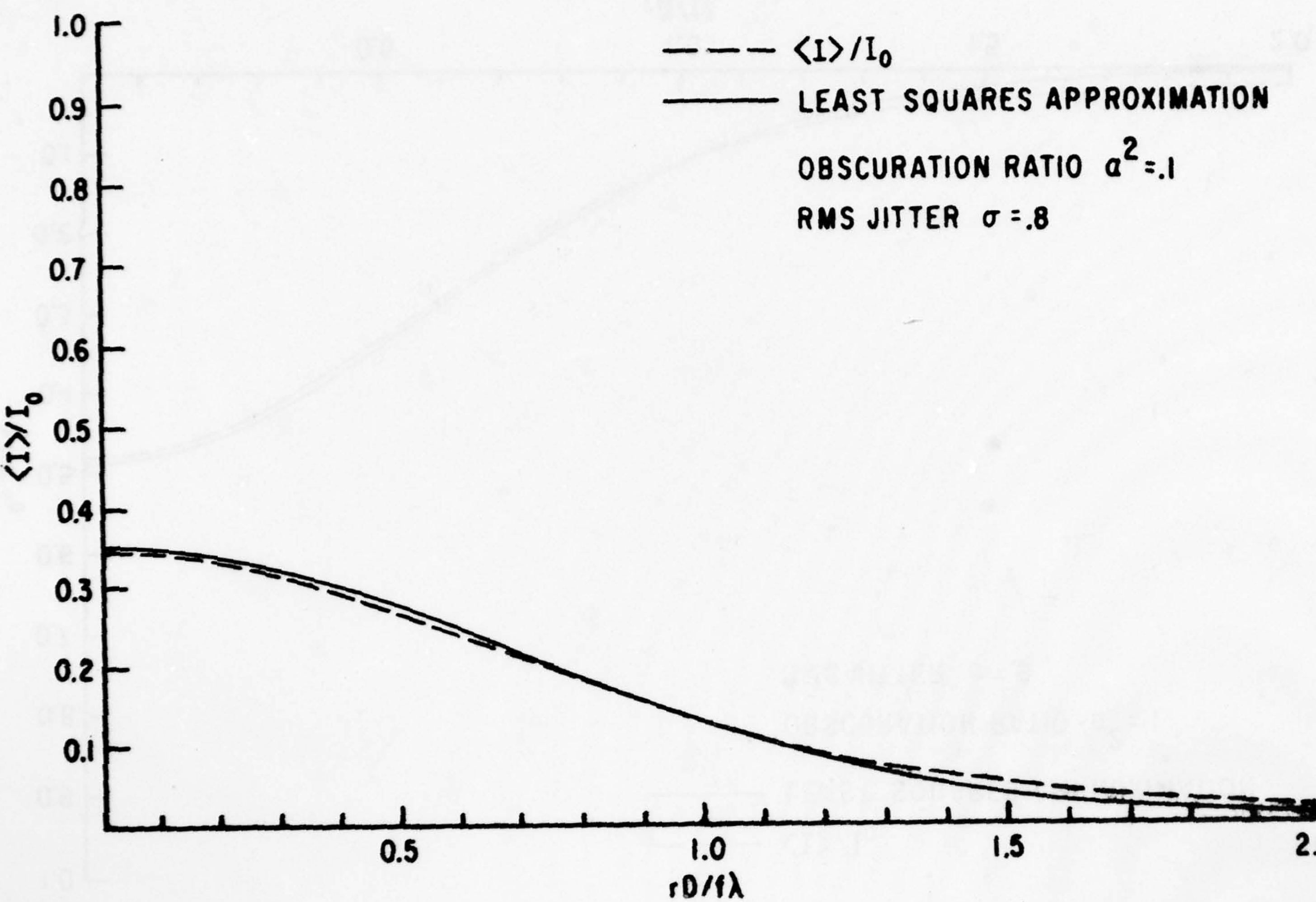


Figure 5(b)-5. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

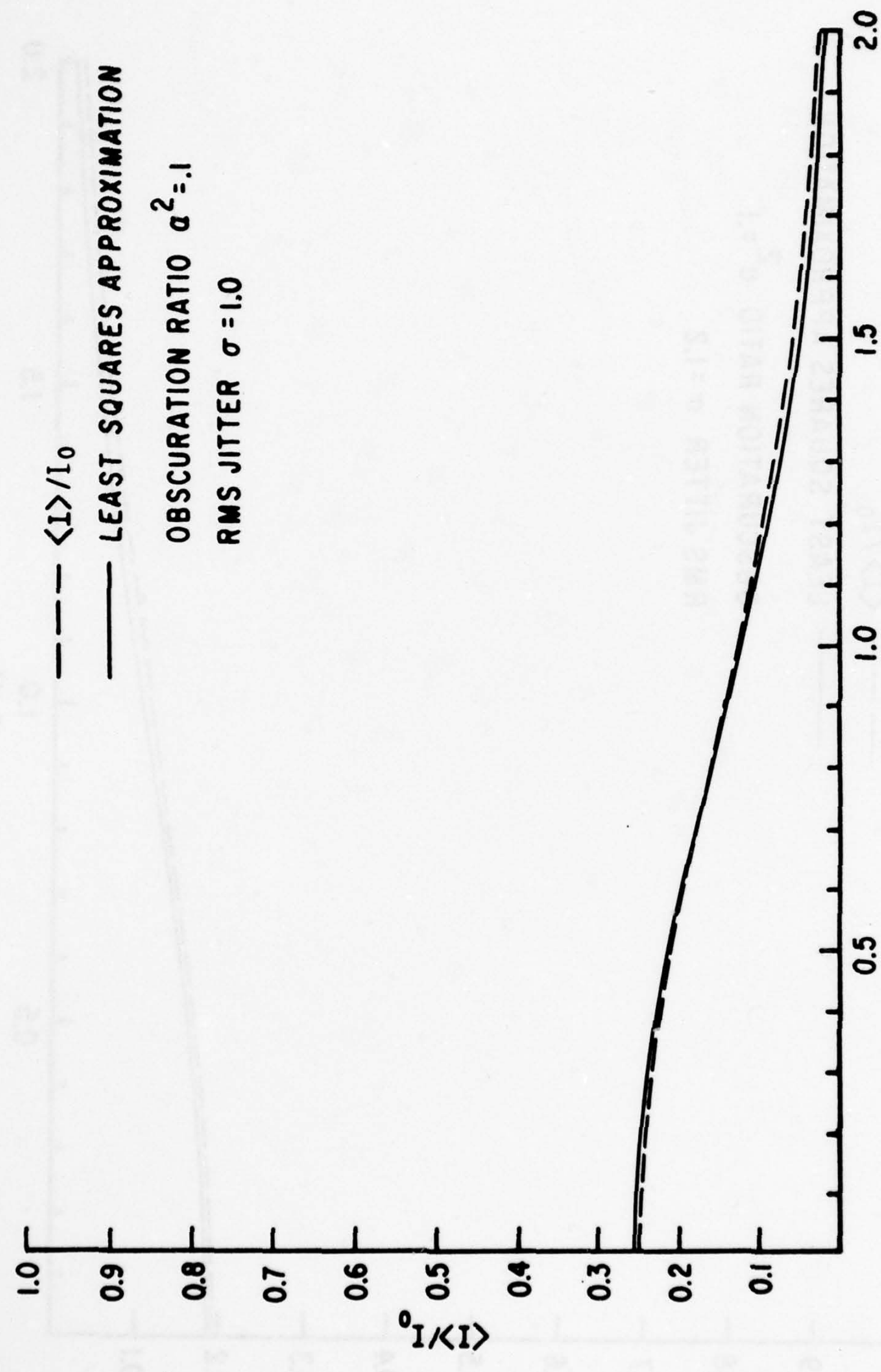


Figure 5(b)-6. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

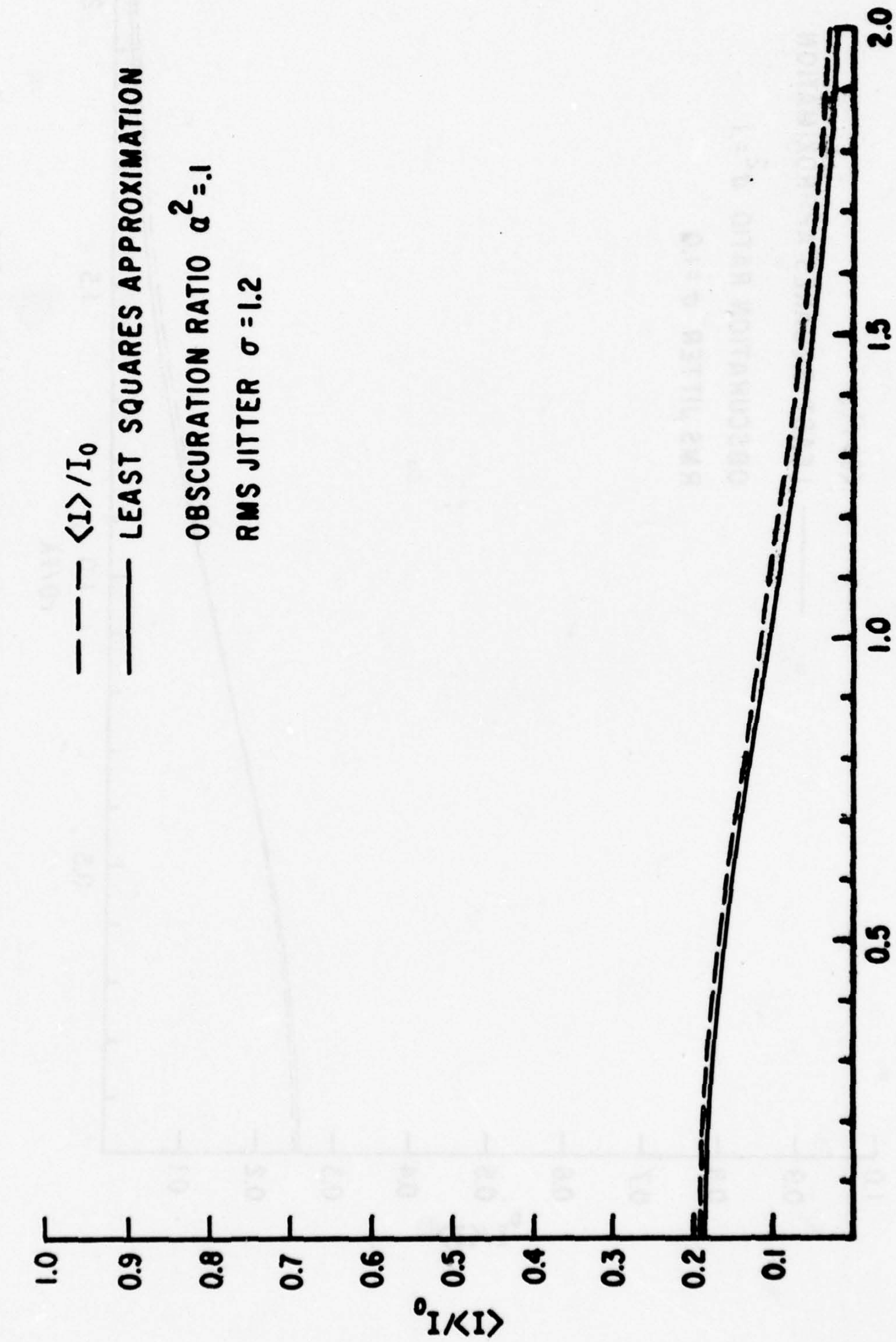


Figure 5(b)-7. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

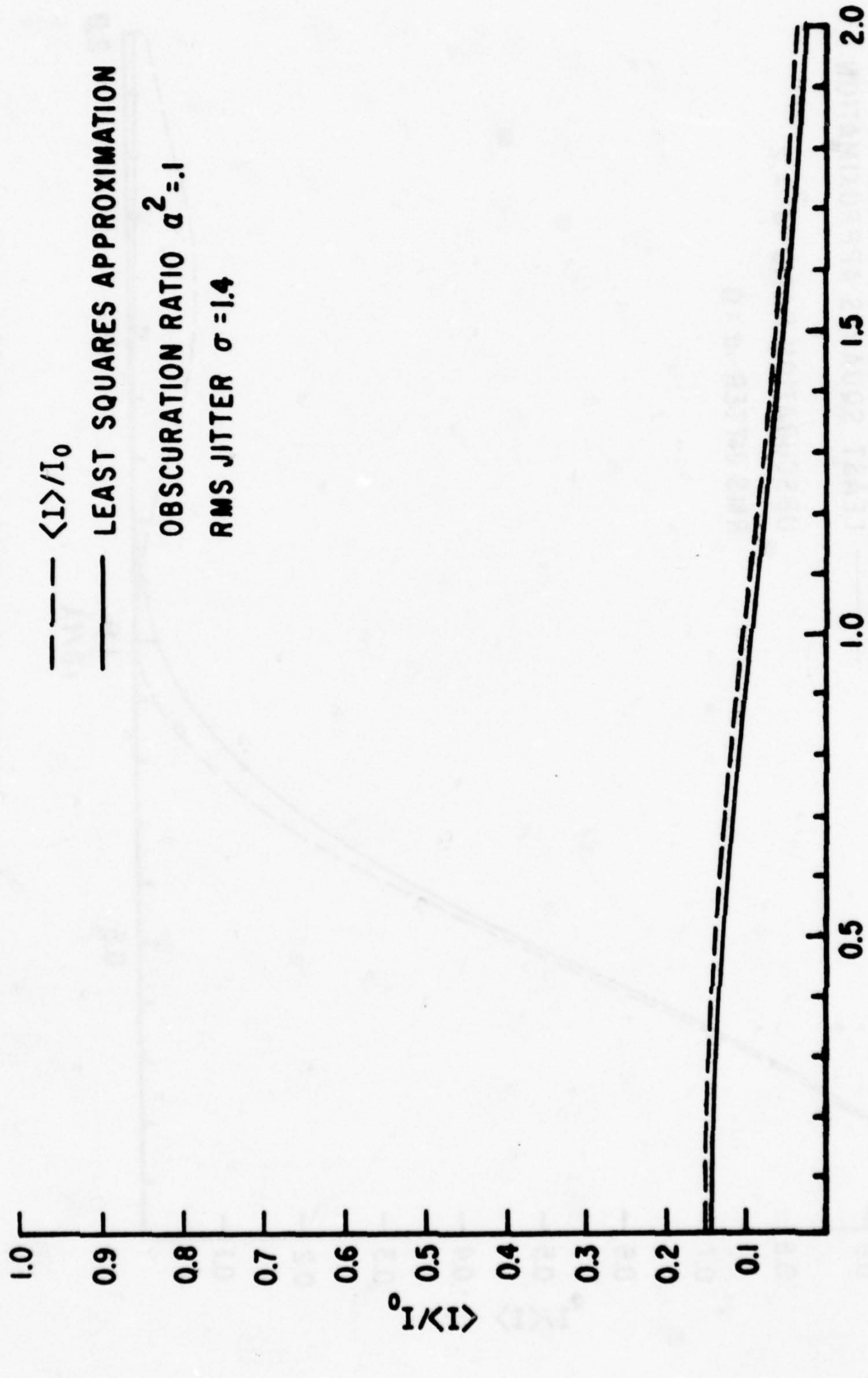


Figure 5(b)-8. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

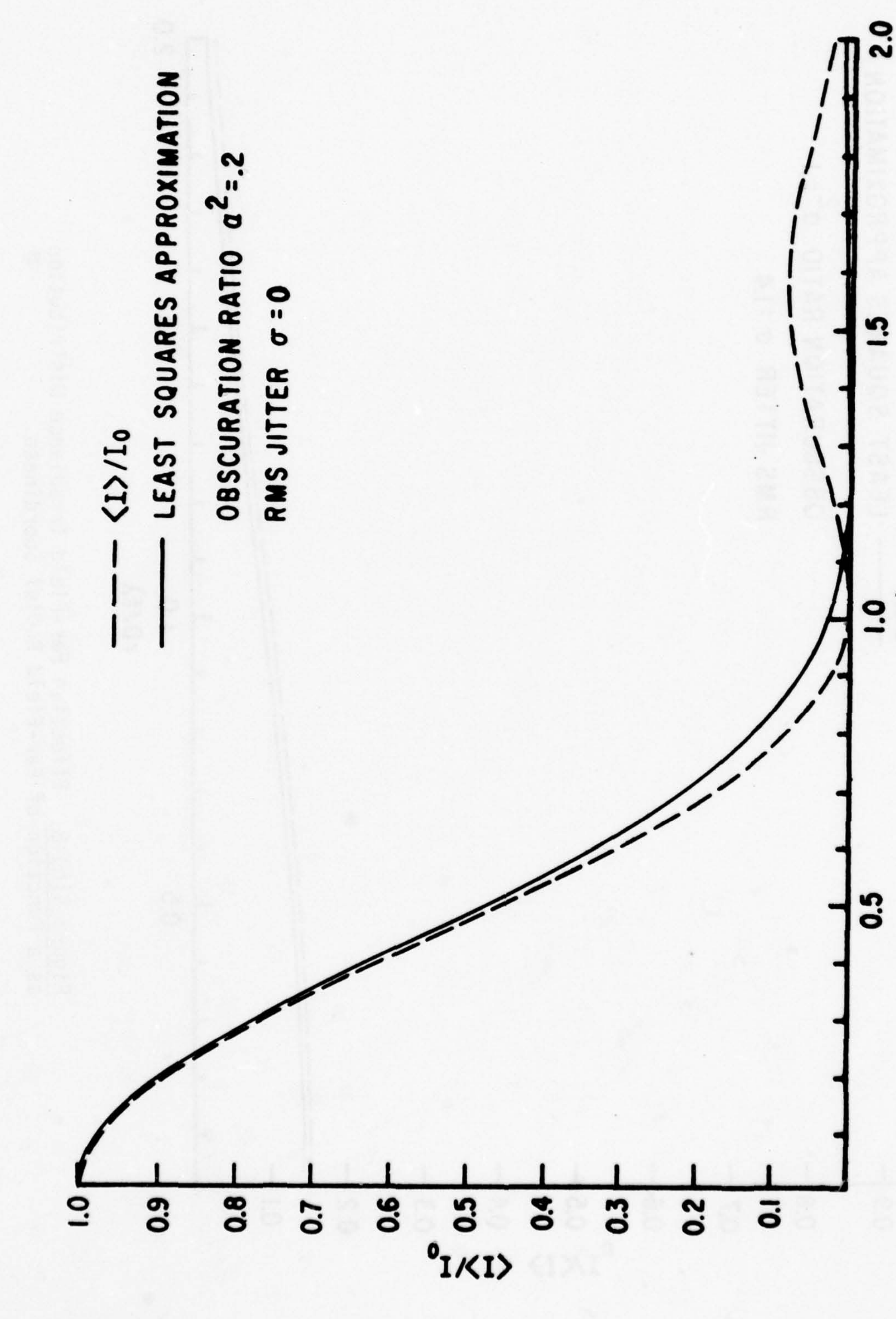


Figure 5(c)-1. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

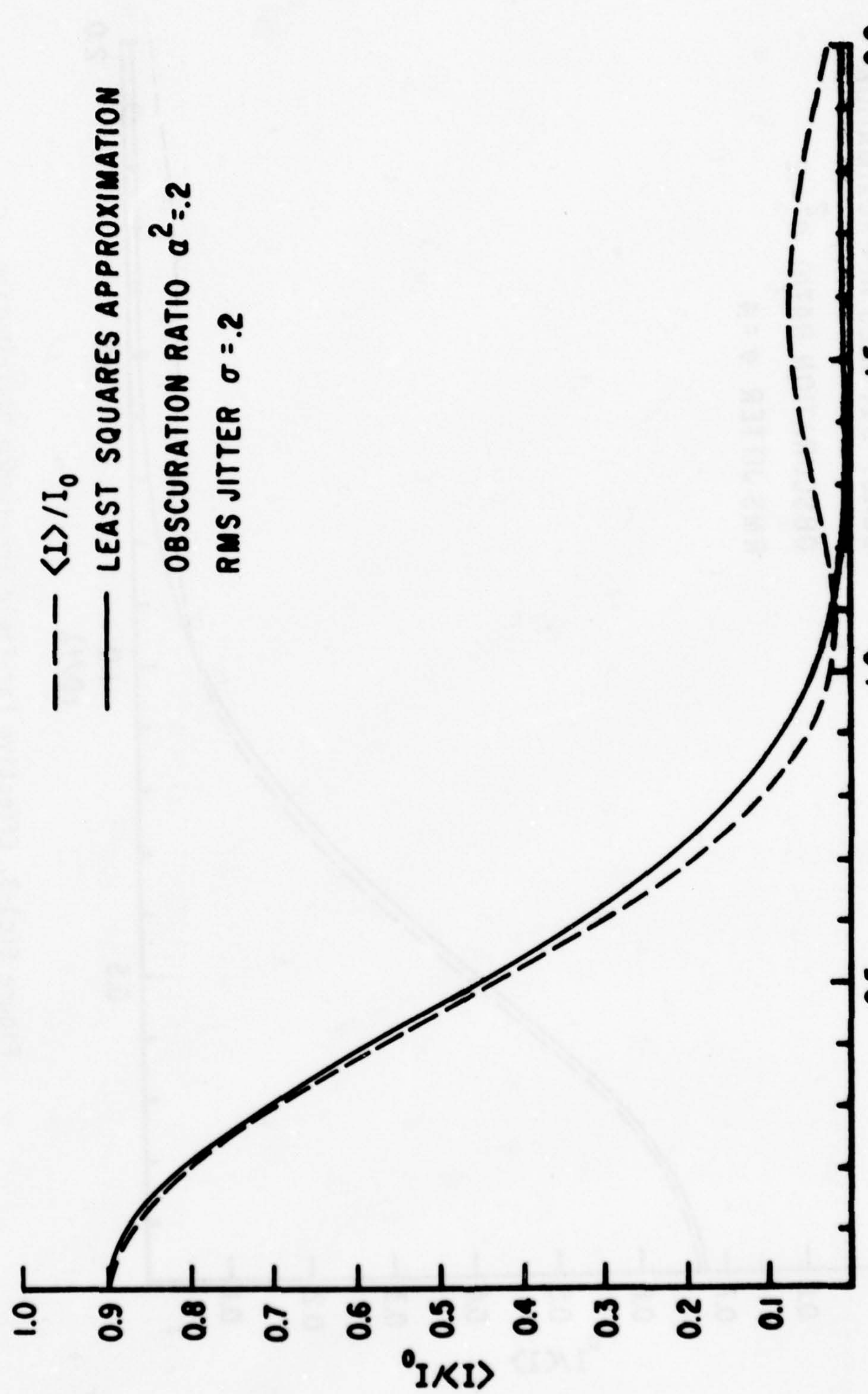


Figure 5(c)-2. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

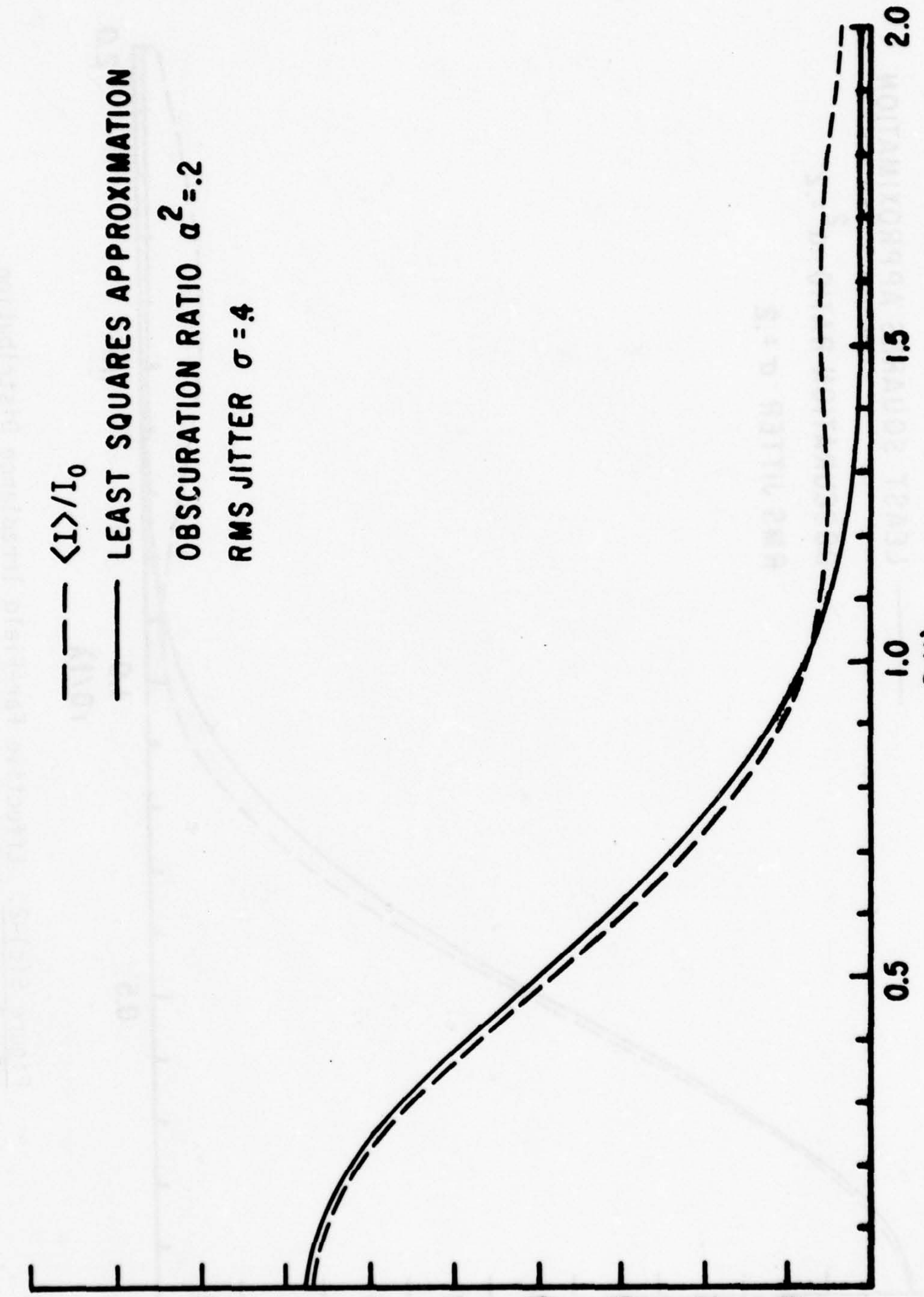


Figure 5(c)-3. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

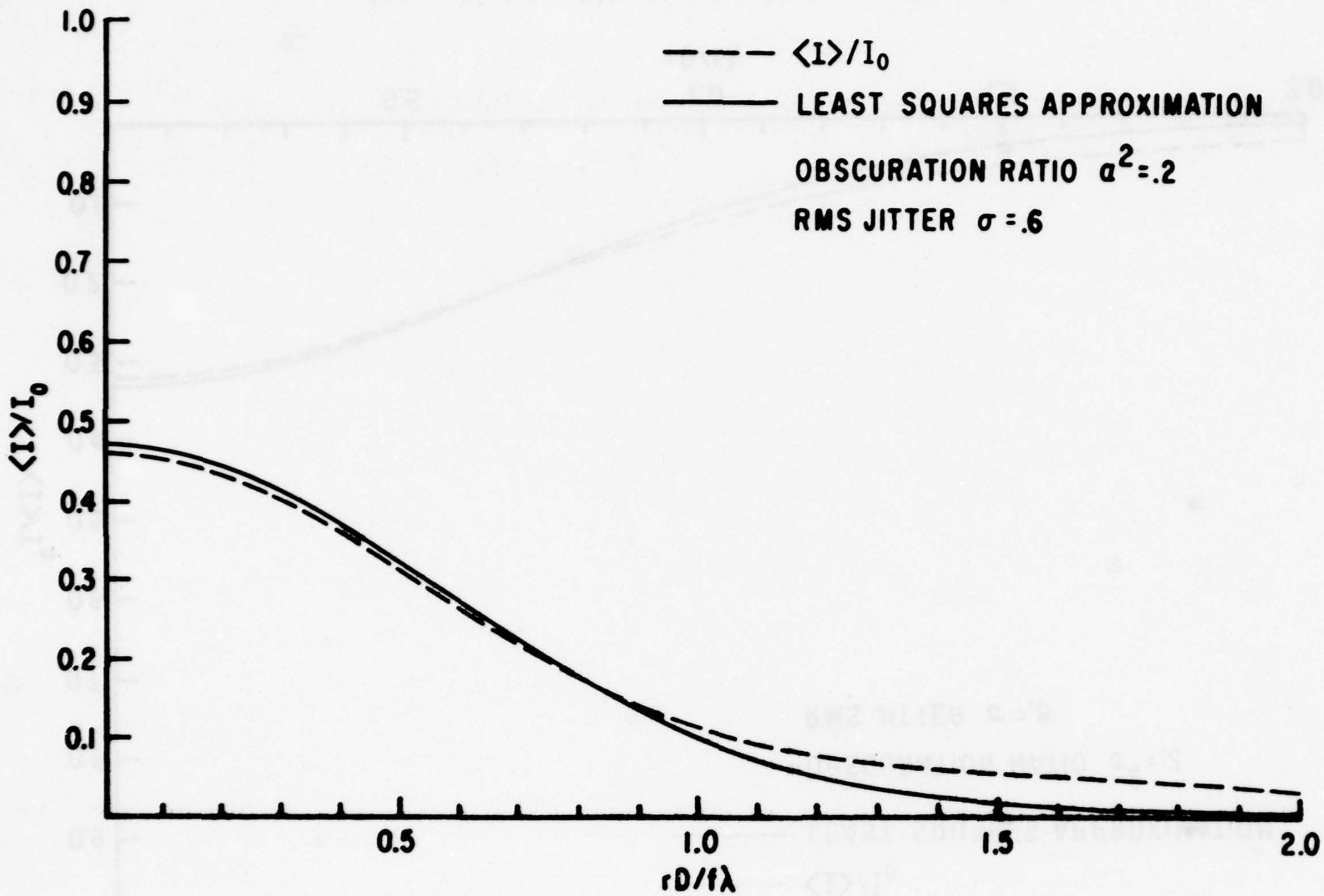


Figure 5(c)-4. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

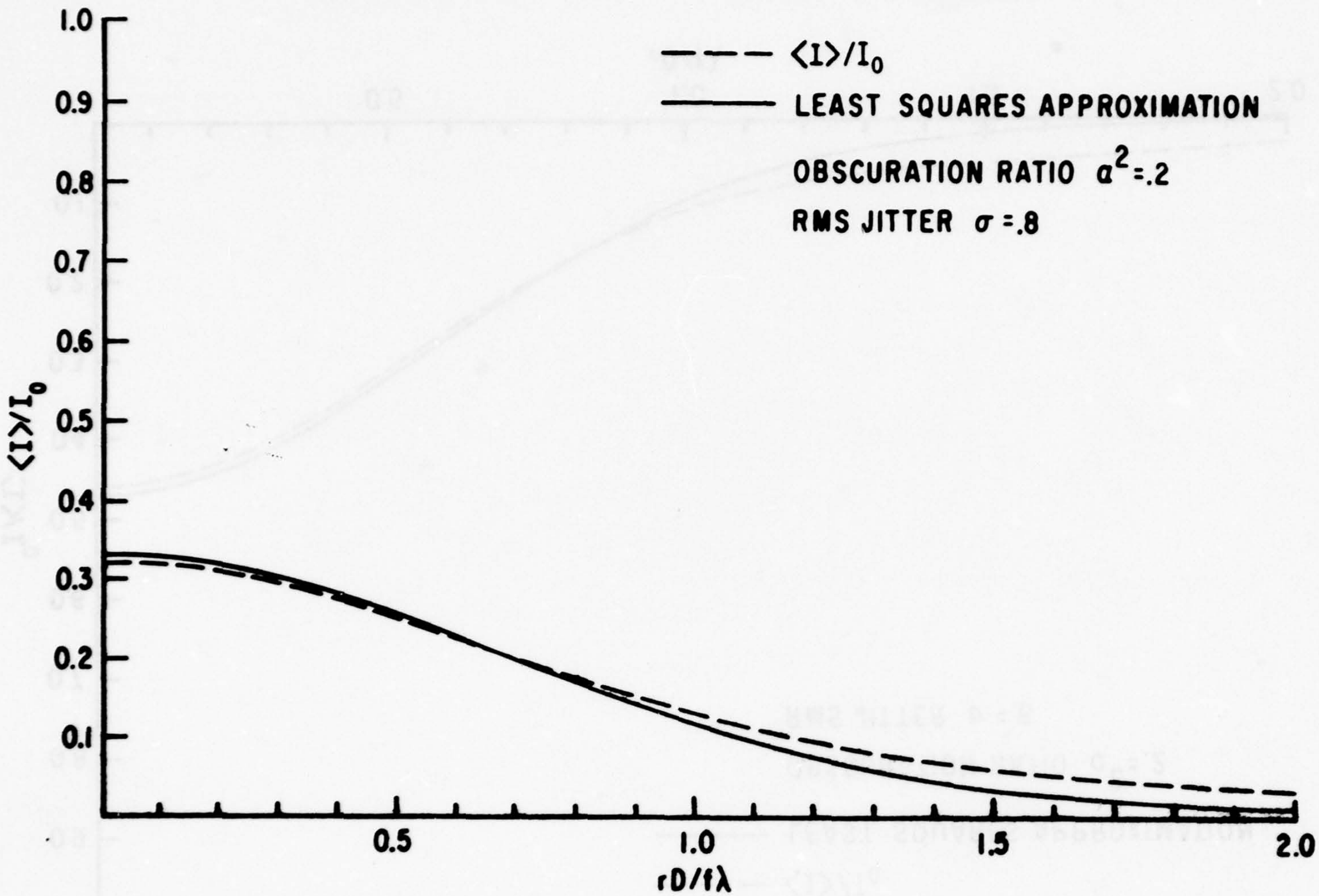


Figure 5(c)-5. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

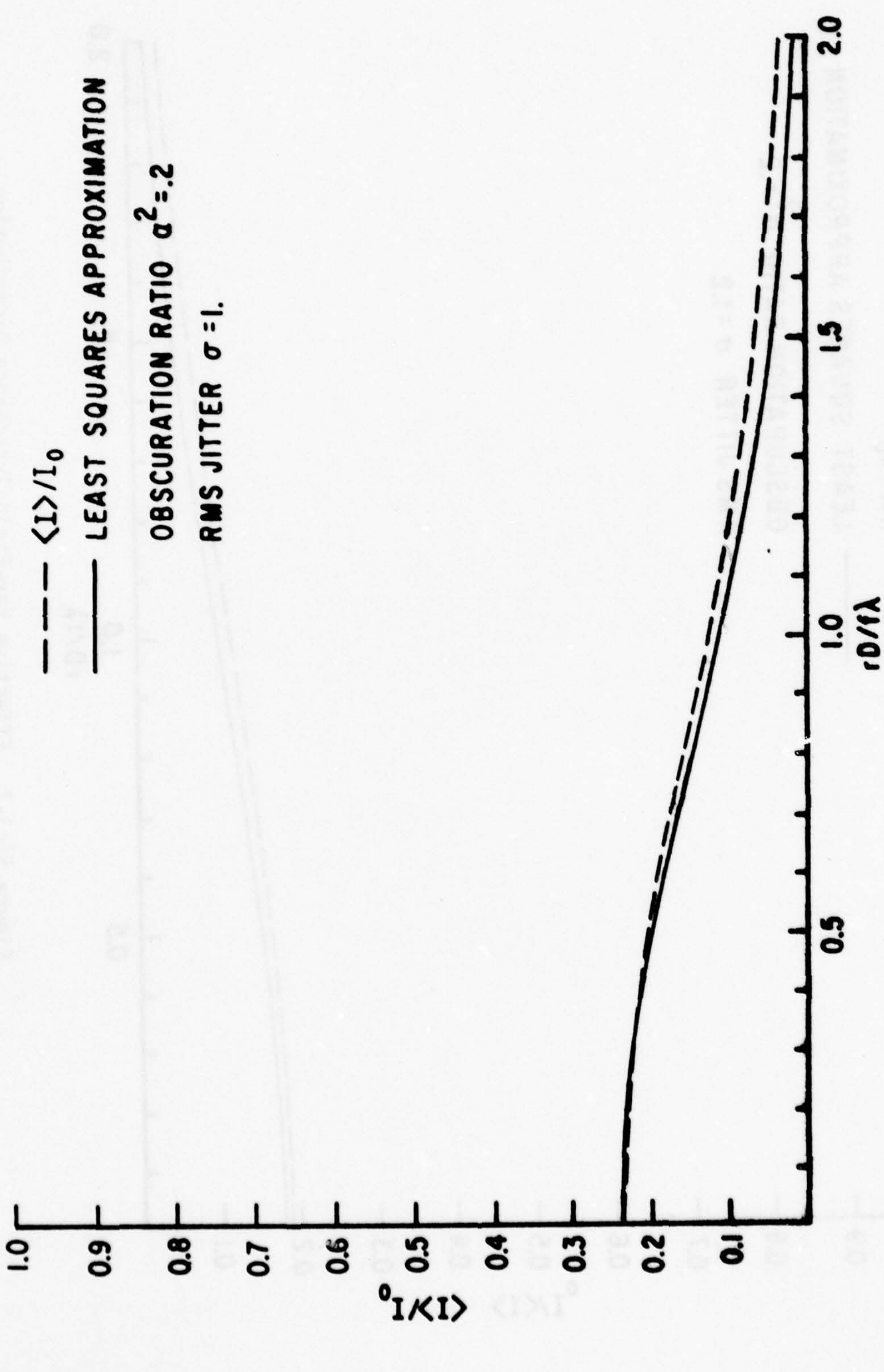


Figure 5(c)-6. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

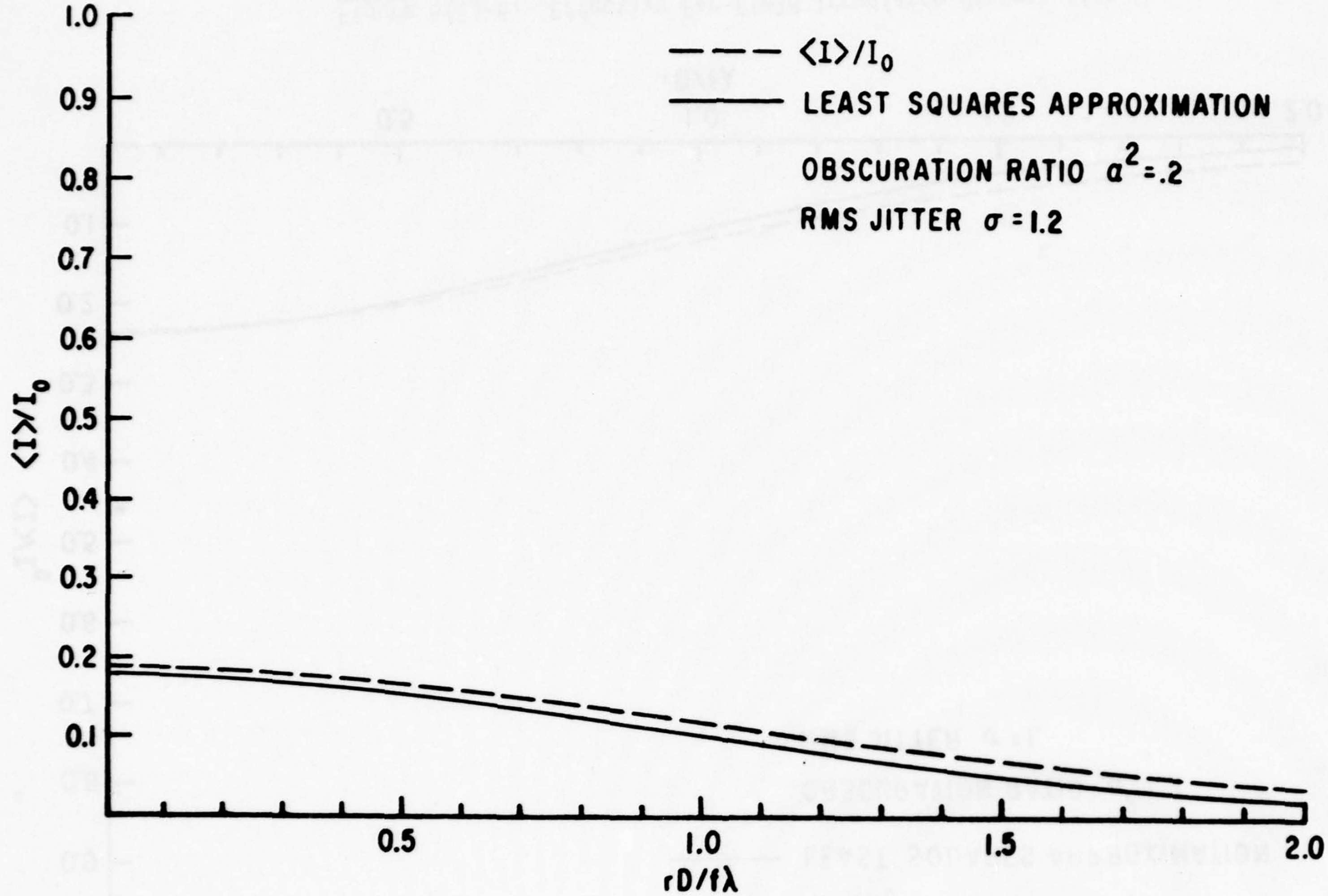


Figure 5(c)-7. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

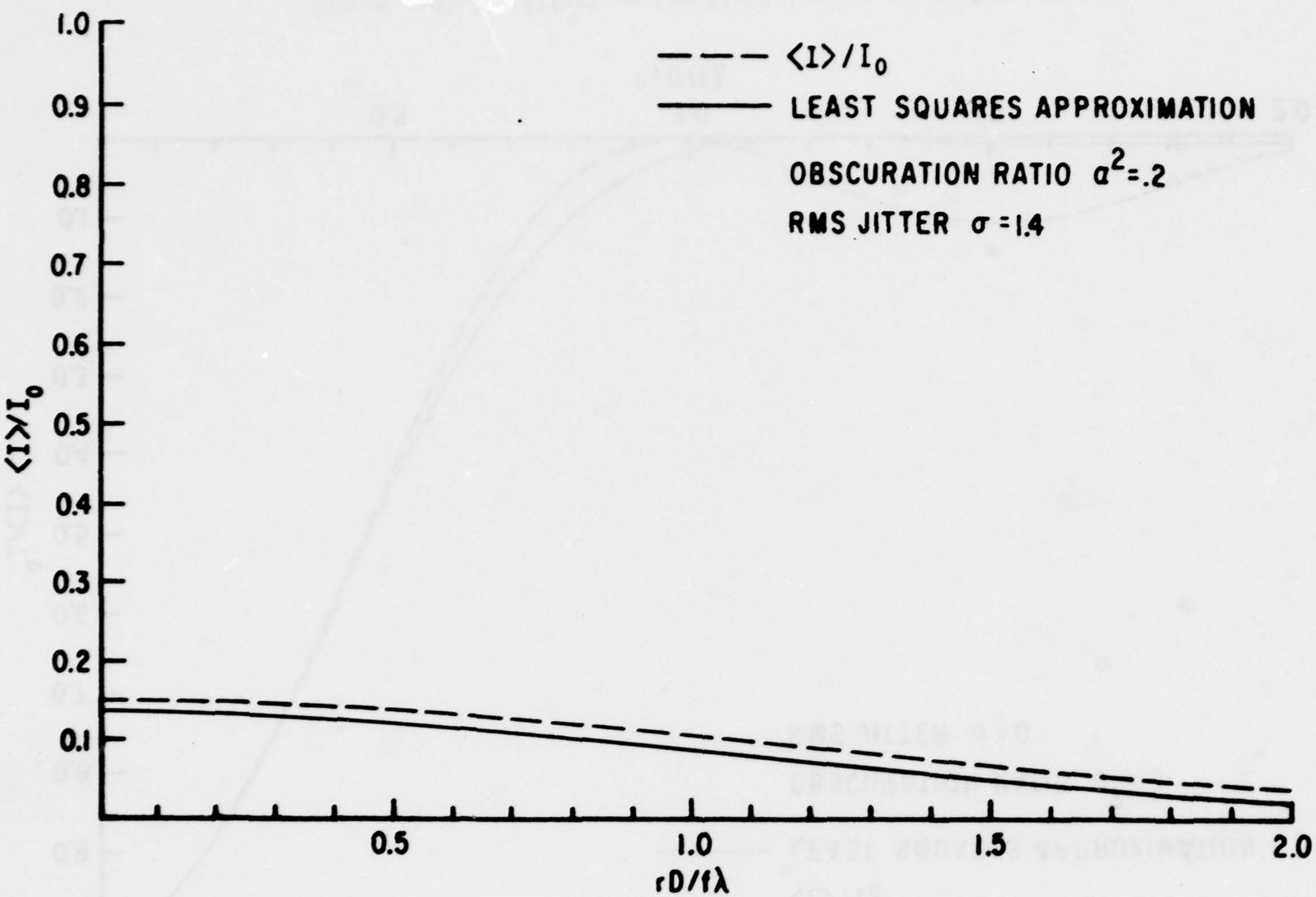


Figure 5(c)-8. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

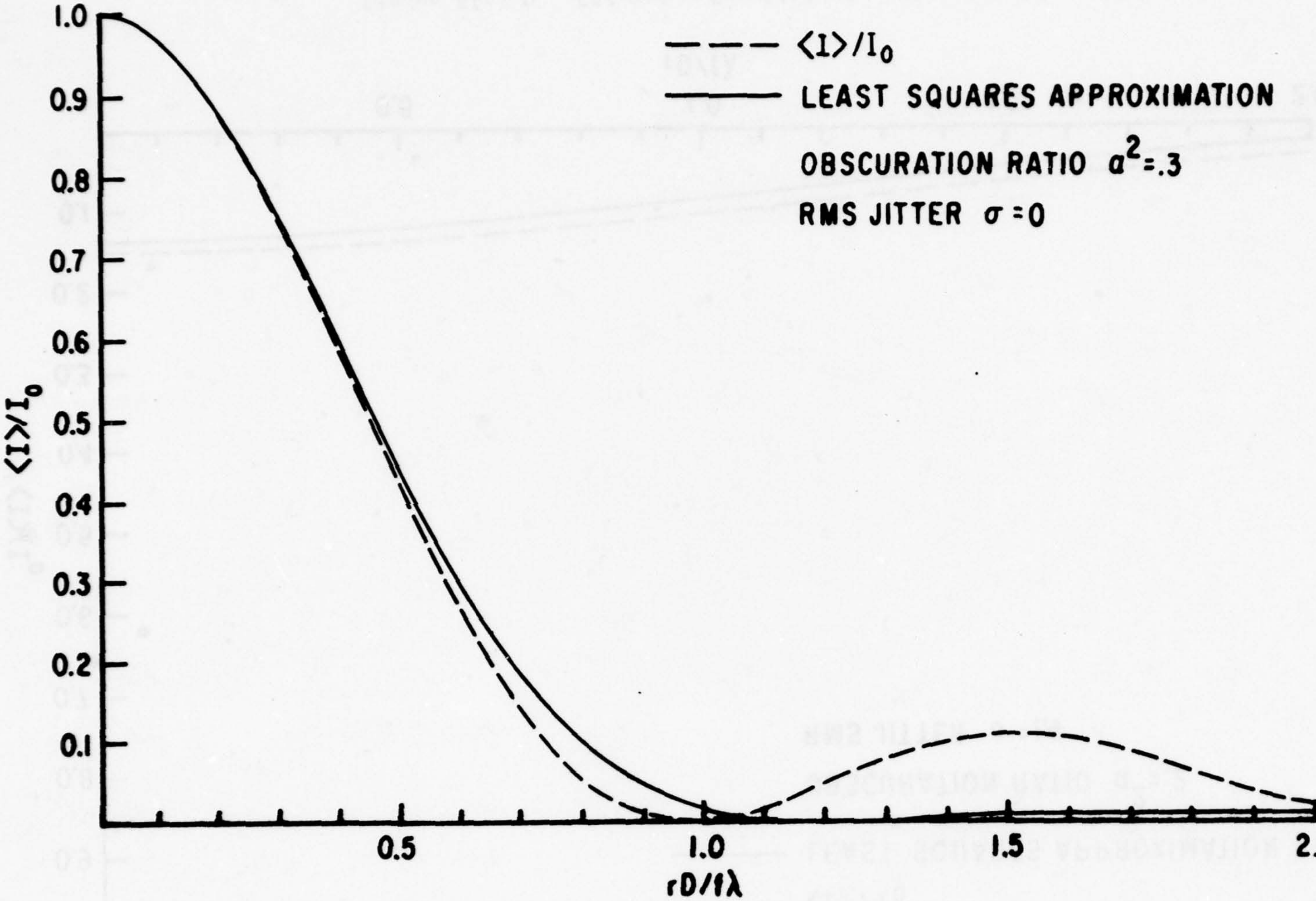


Figure 5(d)-1. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

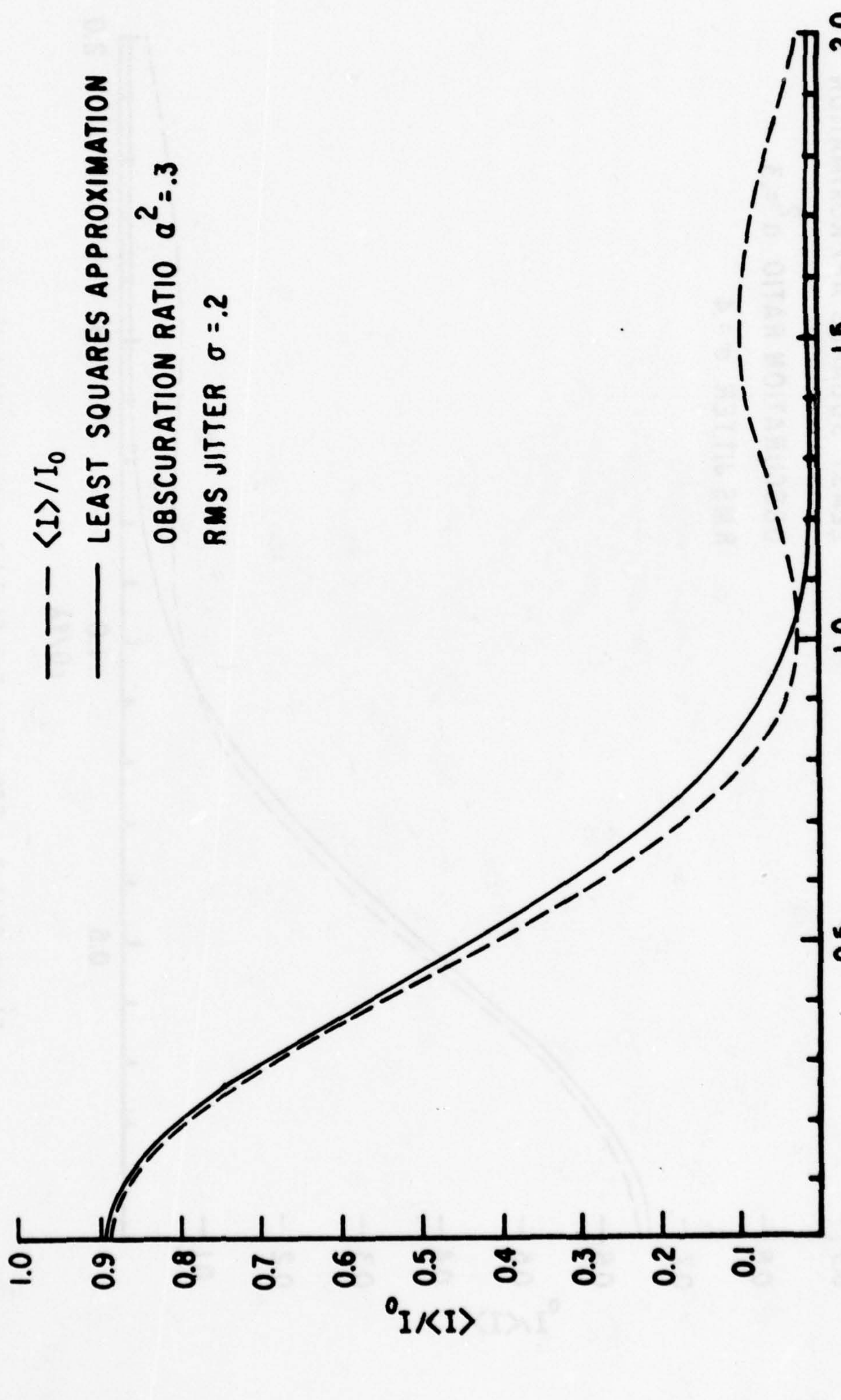


Figure 5(d)-2. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

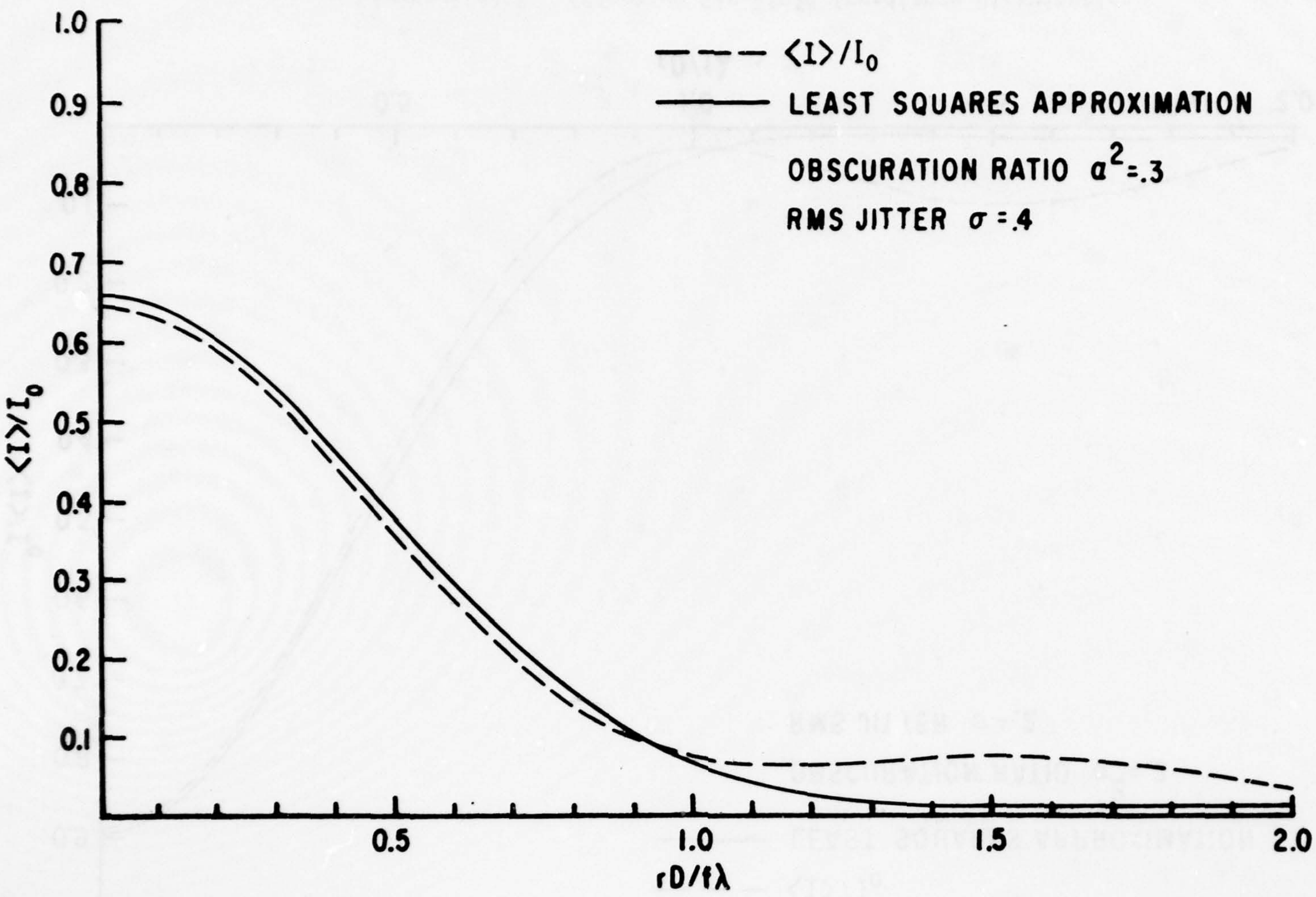


Figure 5(d)-3. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

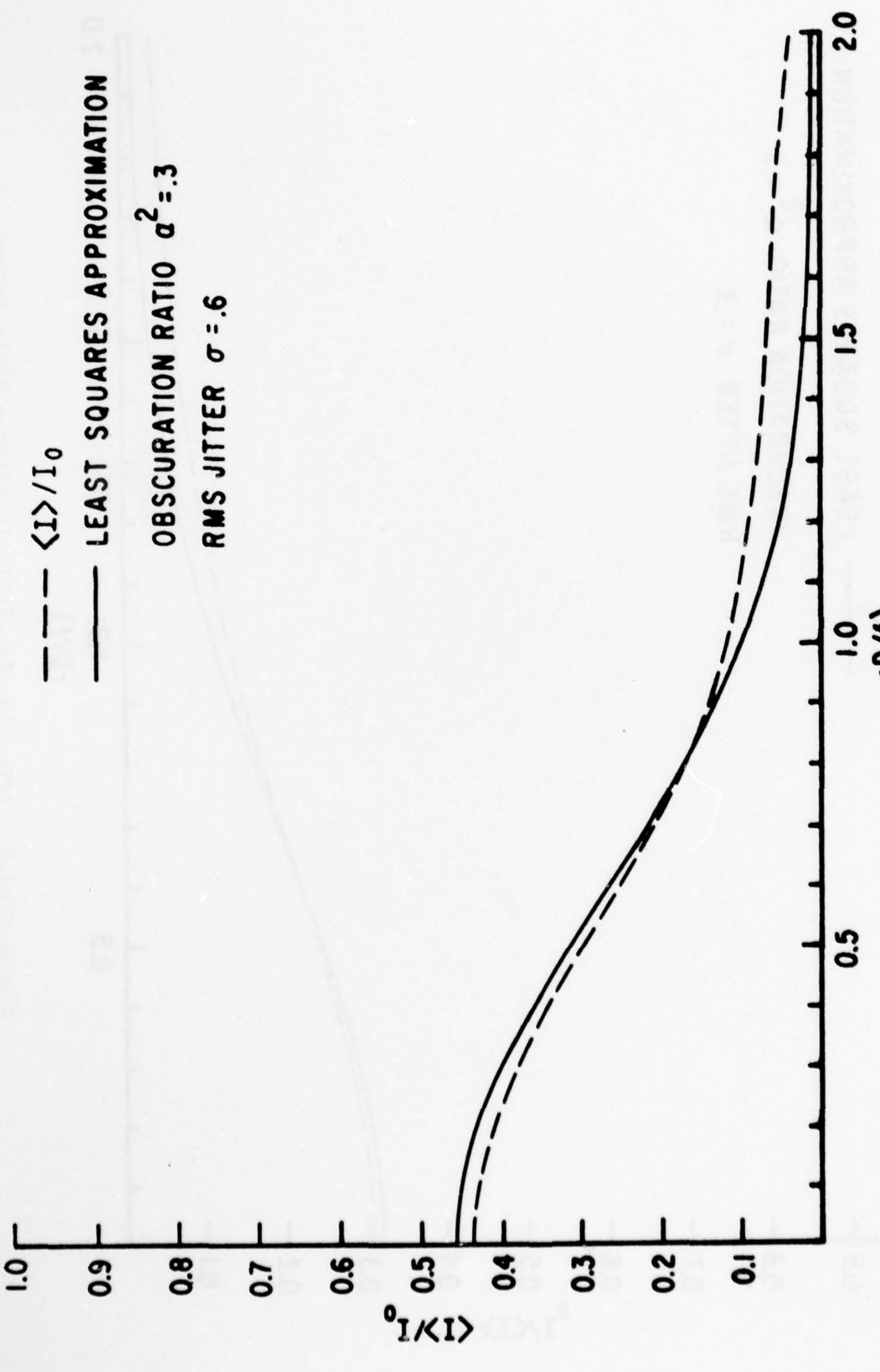


Figure 5(d)-4. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

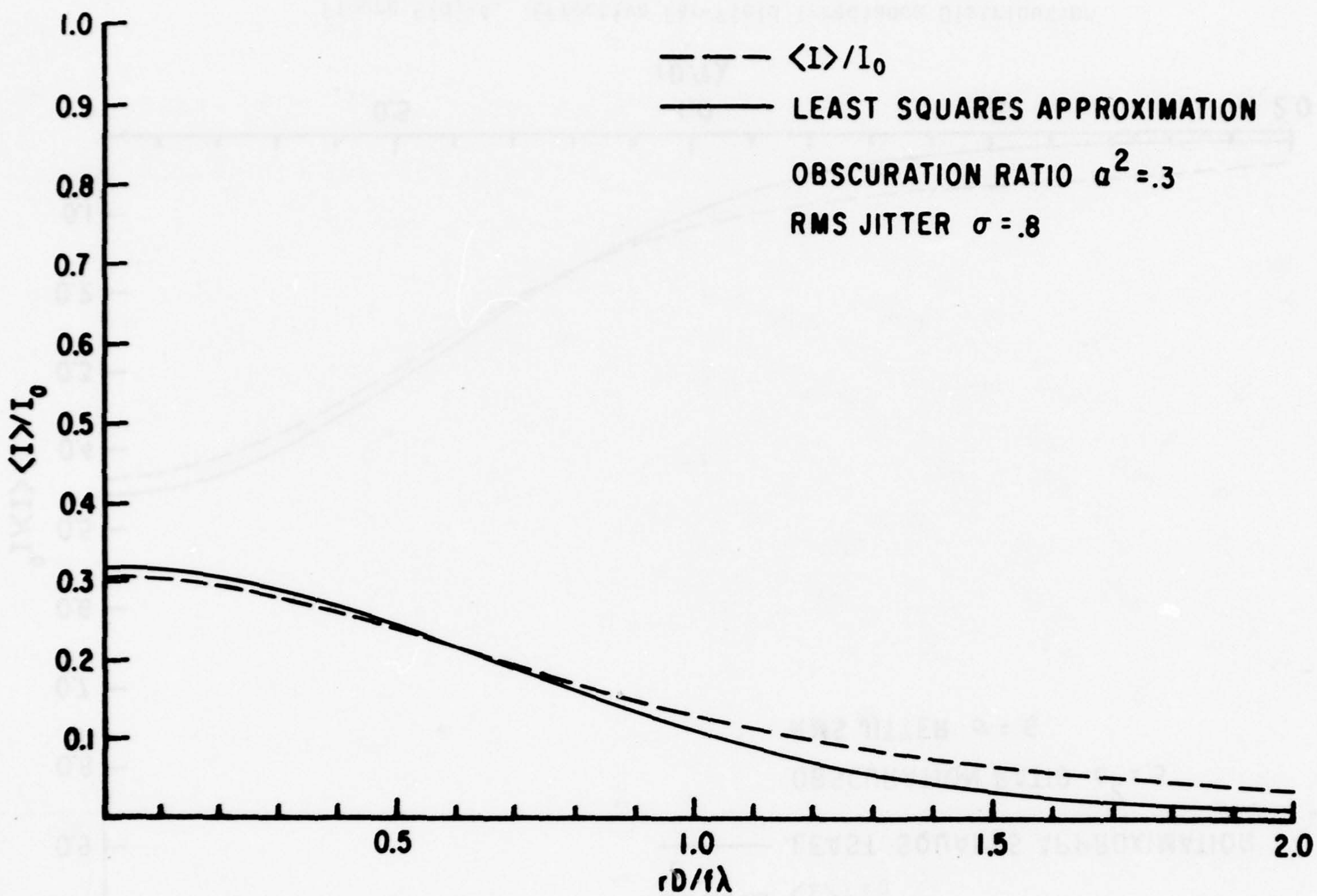


Figure 5(d)-5. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

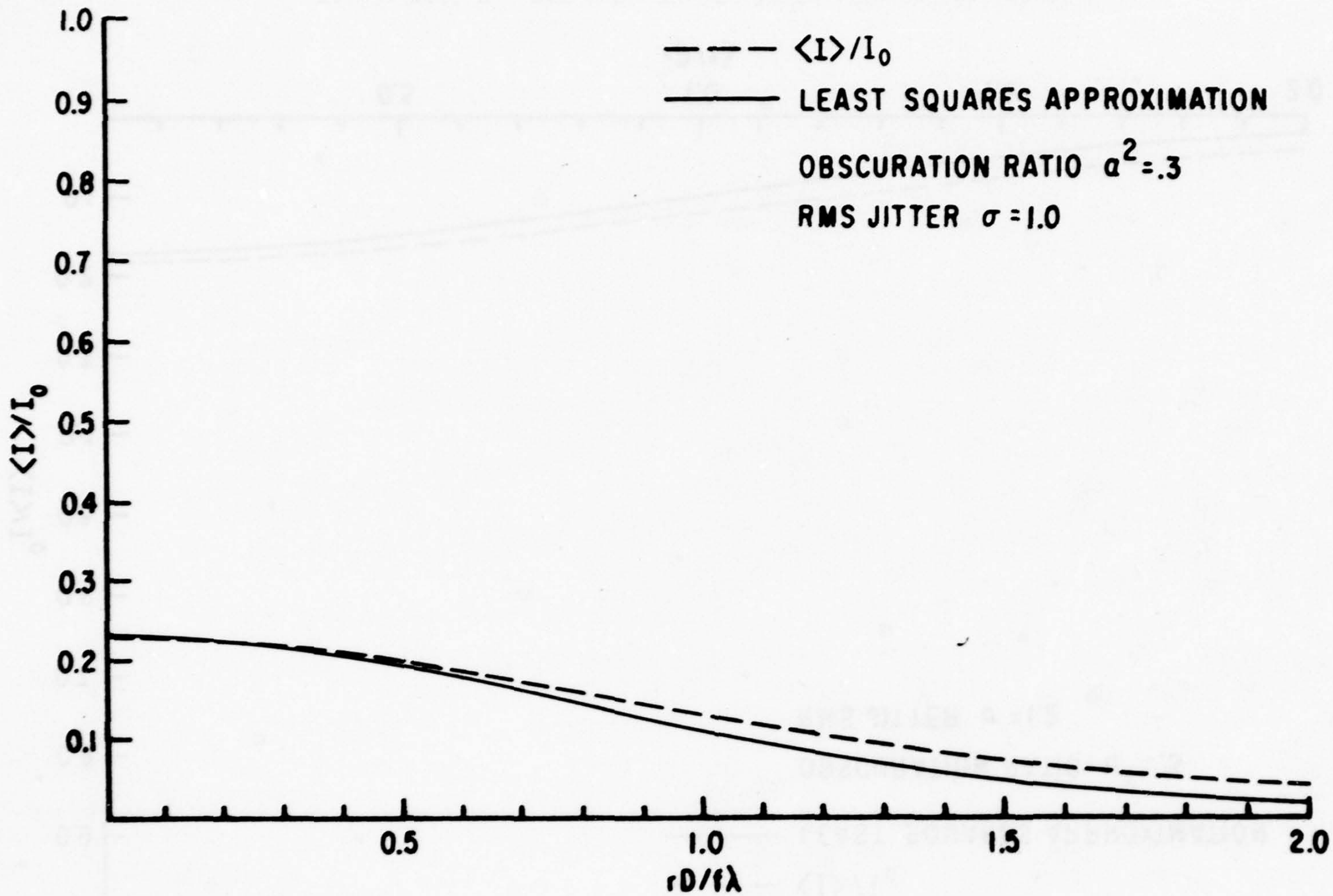


Figure 5(d)-6. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

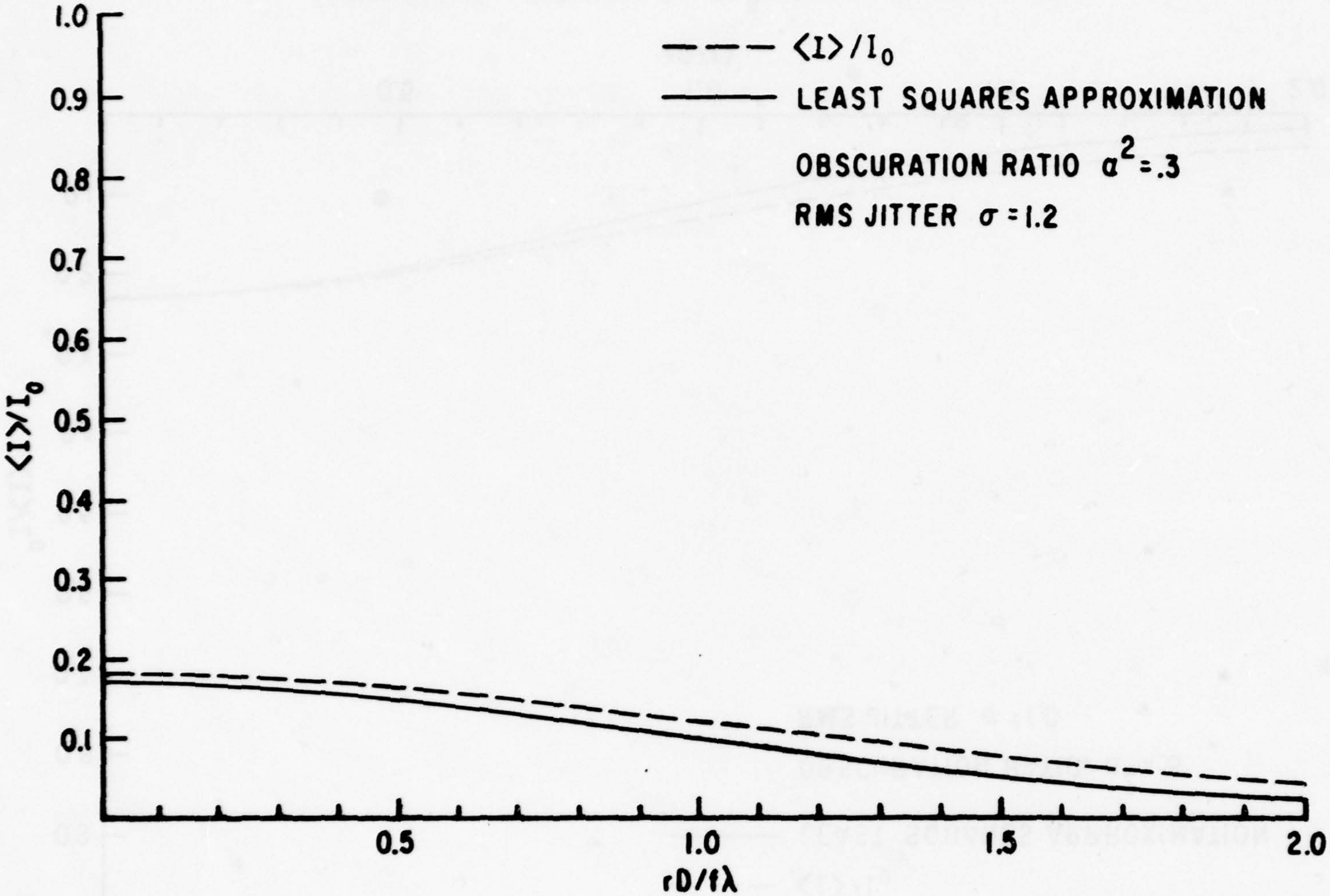


Figure 5(d)-7. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

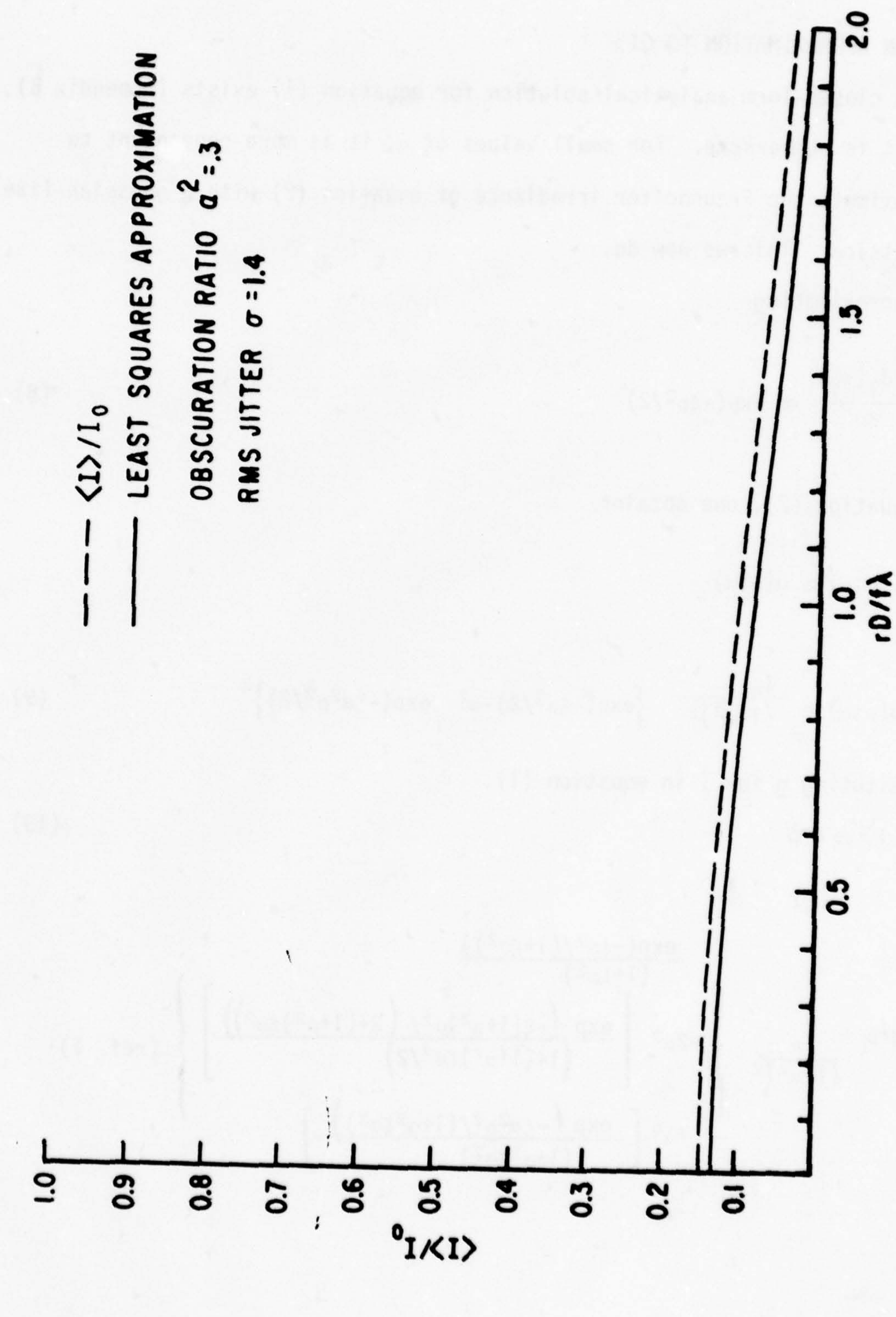


Figure 5(d)-8. Effective Far-Field Irradiance Distribution as a Function of Far-Field Radial Coordinate.

2. AN APPROXIMATION TO $\langle I \rangle$

A closed-form analytical solution for equation (1) exists (Appendix B), but it is cumbersome. For small values of α, it is more convenient to approximate the Fraunhoffer irradiance of equation (2) with a gaussian-like expression. This we now do.

By approximating

$$\frac{2J_1(\pi\rho)}{\pi\rho} \approx \exp(-\xi\rho^2/2) \tag{8}$$

in equation (2), one obtains

$$I(\rho;\alpha) \approx g(\rho;\alpha)$$

where

$$g(\rho;\alpha) = \frac{I_o}{(1-\alpha^2)^2} \left\{\exp(-\xi\rho^2/2) - \alpha^2 \exp(-\xi\alpha^2\rho^2/2)\right\}^2 \tag{9}$$

Substituting g for I in equation (1),

$$\langle I \rangle \approx g*p \tag{10}$$

$$g*p = \frac{I_o}{(1-\alpha^2)^2} \left\{ \begin{array}{l} \dfrac{\exp(-\xi\rho^2/(1+\xi\sigma^2))}{(1+\xi\rho^2)} \\ -2\alpha^2 \left[\dfrac{\exp\left(-\xi(1+\alpha^2)\rho^2/\left(2+(1+\alpha^2)\xi\sigma^2\right)\right)}{\left(1+(1+\alpha^2)\xi\sigma^2/2\right)} \right] \\ +\alpha^4 \left[\dfrac{\exp\left(-\xi\alpha^2\rho^2/(1+\alpha^2\xi\sigma^2)\right)}{(1+\alpha^2\xi\sigma^2)} \right] \end{array} \right\} \quad \text{(ref. 1)}$$

Physically, g represents the interference pattern of two focused gaussian beams whose beam waists are w and w/α, where

$$w^2 = \left(\frac{f\lambda}{D}\right)^2 2/\xi \qquad (11)$$

The parameter ξ is taken to be a function of α only. By setting $\rho = 0$ and least-squares fitting equation (10) to the digital computer solutions of equation (7), one obtains

$$\xi \approx 2.724 - 1.193\alpha^2 + .9353\alpha^4 \qquad (12)$$

Equation (9) is a useful approximation for $\langle I \rangle$ when one wishes to find the effect of jittered, uniformly intense, annular "plane waves" where the annulus has a value of α other than those considered in this paper.

SECTION VI
CONCLUSIONS

Jitter is one major cause of laser beam degradation. In effect it broadens an otherwise diffraction-limited beam, causing a corresponding decrease in peak irradiance and washing out interference fringes.

An increase in jitter usually but not always decreases the time-averaged power transmitted by a circle that is centered at the origin of the Fraunhoffer plane. If the rms gaussian jitter is small by comparison to the normalized radius of the circle, i.e., if

$$\frac{aD}{f\lambda} \gg \sigma \qquad (12)$$

then the effect of jitter upon the power transmitted by a circle of radius a is negligible within a tolerance of a few percent.

From such a criterion it is apparent that as the radius of the circle decreases to zero, it is impossible to have the effect of jitter be negligible. The power transmitted by a tiny circle centered at the point of time-averaged peak irradiance is highly sensitive to jitter. Normalizing the transmitted power by the area of the circle, one has, in the limit of vanishing area, the value of the peak irradiance. Consequently the peak irradiance falls off much faster as jitter increases than does the irradiance farther off axis. Measurements with detectors whose diameters are much smaller than the nominal beam width are quite sensitive to jitter.

From a viewpoint of raw energy transfer, large area receivers are

insensitive to jitter. Based on Figure 2, a circle whose radius, a, is approximately $f\lambda/D$ can tolerate an rms jitter of about .5 $f\lambda/D$ with about 10% loss of transmitted energy. A circle of twice the diameter can tolerate an rms jitter of about 1 $f\lambda/D$. A circle of $a \approx 3\ f\lambda/D$ can tolerate about 1.5 $f\lambda/D$. Hence, a rule of thumb for large area circles is that if the radius of the circle equals a, then an rms jitter of a/2 will induce about 10% loss of power transmitted by the circle. This rule of thumb seems to be fairly independent of the annular obscuration ratio α^2 for $\alpha^2 \lesssim .3$.

From a standpoint of beam quality a given amount of jitter induces a larger drop in central intensity than it does in encircled irradiance. (See figures 4(a) - 4(c).) To anyone concerned with specifying jitter tolerances of a focusing optical system, a specific limitation on the degradation of the central irradiance, say 10% will place a tighter restriction on jitter than the same (10%) degradation limitation on encircled power.

This paper has considered only a few values of the obscuration ratio α^2. By using equation (8) it is easy to estimate the effect of jitter upon the Fraunhoffer irradiance distribution for other obscuration ratios.

REFERENCES

1 Negro, J.E., "Pointing Variance and Beam Degradation Calculations," Laser Digest, AFWL-TR-74-100, Air Force Weapons Laboratory, Kirtland Air Force Base, NM, Spring 1974.

2 Esposito, R., Letter, Proceedings of the IEEE, August 1967.

3 Born, M. and Wolf, E., Principles of Optics, Fourth Edition, Pergamon Press, NY, 1970, p. 441.

4 Gradshteyn, I.S., and Ryzhik, I.M., Tables of Integrals, Series and Products (Fourth Edition), Translated from the Russian by Scripta Technica, Inc., Academic Press, New York City, NY, 1965.

APPENDIX A

APPLICABILITY OF MODELING JITTER OF A FOCUSED BEAM AS A CONVOLUTION

Although it was assumed that jitter caused the Fraunhoffer pattern to undergo simple, unaberrated translation over the entire plane surface, in practive this does not happen. Real jitter causes the focal spot to trace out a path that is not contained in a single plane. Also, no excursion is ever infinitely far from the nominal optical axis. In this article, we ignore all wavefront aberrations and assume our instantaneous irradiance distribution is always given by the Fraunhoffer pattern, equation (2) of the main body of this paper. With this restriction in mind, we address the following two questions: First, what is the effect of assuming that jitter causes simple translation over a plane? Secondly, what is the error in assuming that jitter can cause the beam to wander infinitely far from the optical axis?

A simple model for jitter is depicted below. A stationary beam strikes a wiggling mirror. ϕ_0 represents the line-of-sight off-axis angle to the instantaneous center of the jittering irradiance distribution.

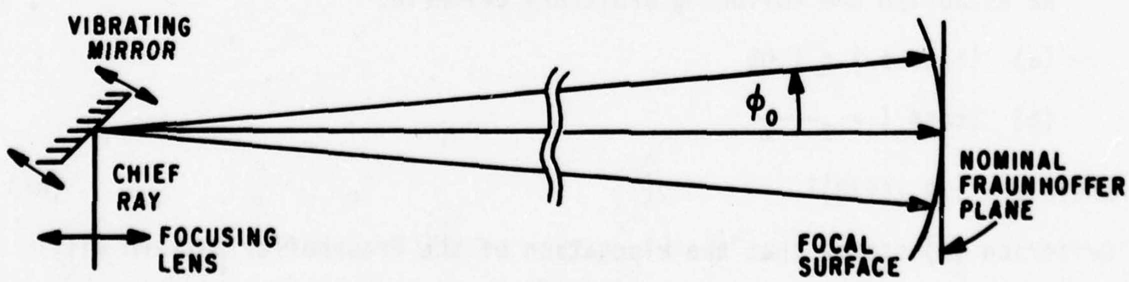


Figure A1. Simple Jitter Model

For large enough angles of jitter, two effects occur. First of all, the beam no longer focuses in the plane it originally focused in. Secondly, even if the depth of focus is sufficient to allow a Fraunhoffer pattern to appear in the original focal plane, distortion occurs. The pattern is elongated because the pattern in the nominal Fraunhoffer plane is really a projection of the pattern along the true focal surface.

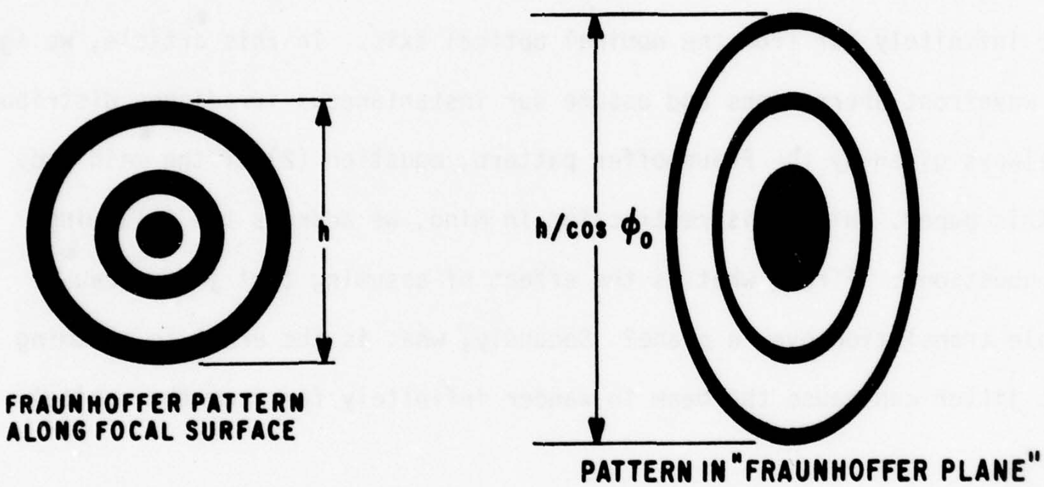


Figure A2. Elongation of Far-Field Pattern due Tilt of Beam by an Amount ϕ_0

We establish the following arbitrary criteria:

(a) $|1/\cos\phi_0| \leq 1.05$

(b) $|\cos\phi_0| \geq \frac{f}{f + \ell}$

where $\ell = 1/2\ \lambda(f/D)^2$ (A1)

Criterion (a) states that the elongation of the Fraunhoffer pattern will not exceed five percent. Criterion (b) states the true location of peak

far-field irradiance will always be within the depth of focus of the nominal focal plane. The equation for depth of focus is derived strictly from considerations of decrease in on-axis irradiance as one moves in and out of focus. It does not describe the misfocus one can tolerate and still see higher order Fraunhoffer fringes.

Let ϕ_{max} be the upper limit of angular jitter. Relating (a) to (b)

$$\frac{f + \ell}{f} \leq 1.05$$

and substituting equation (1) for ℓ, we obtain equation (A2).

$$\frac{\lambda f}{D^2} \leq 0.1 \tag{A2}$$

Typically, the diameter of a Fraunhoffer pattern is on the order of 2 $f\lambda/D$. Therefore, whenever one is far enough away such that the diameter of the Fraunhoffer pattern equals or exceeds a fifth of the diameter of the focusing aperture, and for

$$\phi_{max} \leq \cos^{-1}\left(\frac{1}{1.05}\right) \tag{A3}$$

this paper considers angular-jitter-induced focal spot wander to be confined to a plane, and jitter-induced beam elongation is considered negligible.

In this simplified model of jitter we have considered only angular jitter. We disregard displacements due to pure translational jitter on the basis that the effects of pure translational jitter are limited by the distance which the diffracting aperture can slide back and forth. Usually this distance is much smaller than the diameter of the aperture. Since, by equation (A2), we have already assumed that the size of the focused beam is

at least as large as a fifth of the aperture, pure translational jitter displacements will be small compared to the size of the focused beam whenever they are small compared to the aperture diameter.

We now seek to know the angles of jitter within which our convolution model of jitter is applicable. Let us arbitarily stipulate that ϕ_{max} in equation (A3) is related to σ by

$$\phi_{max} > 2\sigma\ (\lambda/D) \tag{A4}$$

where $\sigma(\lambda/D)$ denotes the rms angular jitter. That is, we allow for the possibility of infinite jitter displacements but the time-averaged probability that the center of the jittered Fraunhoffer pattern is not more than ϕ_{max} radians away from the optical axis is almost unity. That is,

$$P(\phi_o \le \phi_{max}) = P(\rho_o \le 2\sigma) = 0.98$$

Therefore, effects of jitter are assumed negligible for all angles of jitter greater than ϕ_{max}. It is considered allowable to integrate (1) to the limit $\rho_o \to \infty$, although physically there is an upper bound on ρ_o.

In this paper the maximum value of σ is 3.00. Substituting this value of σ into inequality (A4) and relating that to inequality (A3), we infer that all calculations in this paper are a good model when

$$\lambda/D < \frac{1}{6} \cos^{-1}\left(\frac{1}{1.05}\right) \tag{A5}$$

This relation along with relation (A2)

$$\frac{\lambda f}{D^2} \ge 0.1 \tag{A2}$$

are the two basic restrictions which limit the applicability of results presented in this paper.

APPENDIX B

CLOSED-FORM SOLUTION TO EQUATION (1)

Closed-form analytical solutions for $\langle I \rangle$ exist. In the form presented here, they involve infinite sums of Laguerre polynomials. Define f, g, h, ρ, and $(\theta-\theta_0)$ as follows:

$$f(\rho) = \frac{4\, f_0}{(1-\alpha^2)^2} \left\{ \frac{J_1(\pi\rho)}{\pi\rho} - \alpha^2 \frac{J_1(\pi\alpha\rho)}{\pi\alpha\rho} \right\}^2 \tag{1}$$

$$g(\rho) = \frac{e^{-\rho^2/\sigma^2}}{\pi\, \sigma^2} \tag{2}$$

$$h = f*g = \frac{1}{\pi\sigma^2} \iint_{-\infty}^{\infty} f\left(\sqrt{x_0^2+y_0^2}\right) \exp\left[- \frac{(x-x_0)^2+(y-y_0)^2}{\sigma^2} \right] dx_0\, dy_0 \tag{3}$$

$$= \frac{1}{\pi\sigma^2}\, e^{-\left(\frac{x^2+y^2}{\sigma^2}\right)} \iint_{-\infty}^{\infty} F\left(\sqrt{x_0^2+y_0^2}\right) e^{-\left(\frac{x_0^2+y_0^2}{\sigma^2}\right)} e^{-\frac{2xx_0+2yy_0}{\sigma^2}} dx_0\, dy_0$$

$$\rho^2 \equiv x^2 + y^2 \quad , \rho_0^2 \equiv x_0^2 + y_0^2 \tag{4}$$

$$\rho\rho_0 \cos(\theta-\theta_0) \equiv xx_0 + yy_0 \tag{5}$$

$$I_0(z) = \frac{1}{2\pi} \int_0^{2\pi} e^{z\sin(\theta-\theta_0)}\, d\theta = \text{Modified Bessel Function of the First Kind} \tag{6}$$

It follows that

$$h(\rho) = \frac{2}{\sigma^2} e^{-\rho^2/\sigma^2} \int_0^\infty f(\rho_0)\, e^{-\rho_0^2/\sigma^2}\, I_0\left(\frac{2\rho\rho_0}{\sigma^2}\right) \rho_0\, d\rho_0 \tag{7}$$

$$= \frac{2}{\pi^2\sigma^2}\left(e^{-\rho^2/\sigma^2}\right)\frac{4 f_0}{(1-\alpha^2)^2}\Bigg\{\int_0^\infty \rho_0^{-1} J_1^2(\pi\rho_0)\, e^{-\rho_0^2/\sigma^2}\, I_0\left(\frac{2\rho\rho_0}{\sigma^2}\right) d\rho_0$$

$$-2\alpha \int_0^\infty \rho_0^{-1} J_1(\pi\rho_0)\, J_1(\pi\alpha\rho_0)\, e^{-\rho_0^2/\sigma^2}\, I_0\left(\frac{2\rho\rho_0}{\sigma^2}\right) d\rho_0$$

$$+ \alpha^2 \int_0^\infty \rho_0^{-1} J_1^2(\pi\alpha\rho_0)\, e^{-\rho_0^2/\sigma^2}\, I_0\left(\frac{2\rho\rho_0}{\sigma^2}\right) d\rho_0\Bigg\}$$

From equation 8.447(1) in Reference 5,

$$I_0(\xi\,\rho_0) = \sum_{p=0}^{\infty} \frac{(\xi/2)^{2p}}{(p!)^2} (\rho_0)^{2p} \quad (1) \tag{8}$$

Letting $\xi = 2\rho/\sigma^2$, (9)

Eqn (7) becomes

$$h(\rho) = \frac{8}{\pi^2\sigma^2}\frac{f_0}{(1-\alpha^2)^2} e^{-\rho^2/\sigma^2} \sum_{p=0}^{\infty} \frac{(\xi/2)^{2p}}{(p!)^2} \tag{10}$$

$$\times \Bigg\{\int_0^\infty \rho_0^{2p-1} J_1^2(\pi\rho_0)\, e^{-\rho_0^2/\sigma^2}\, d\rho_0$$

$$- 2\alpha \int_0^\infty \rho_0^{2p-1} J_1(\pi\rho_0)\, J_1(\pi\alpha\rho_0)\, e^{-\rho_0^2/\sigma^2}\, d\rho_0$$

$$+ \alpha^2 \int_0^\infty \rho_0^{2p-1} J_1^2(\pi\alpha\rho_0)\, e^{-\rho_0^2/\sigma^2}\, d\rho_0\Bigg\}$$

We now define $V(a,b,p;\sigma)$.

$$V(a,b;p;\sigma) \equiv \int_0^\infty \rho_0^{2p-1} J_1(a\rho_0)\, J_1(b\rho_0)\, e^{-\rho_0^2/\sigma^2} d\rho_0 \tag{11}$$

Using equation 6.633(1) in Reference 5, this equation becomes

$$V(a;b;\rho;\sigma) = \frac{a\,b\,\sigma^{2(p+1)}}{8} \sum_{m=0}^{\infty} \frac{(m+p)!\,(-a^2\sigma^2/4)^m}{m!\,(m+1)!} F(-m,-m-1;2;\frac{b^2}{a^2}) \tag{12}$$

$$h(\rho) = \frac{8}{\pi^2\sigma^2} \frac{f_0}{(1-\alpha^2)^2} e^{-\rho^2/\sigma^2} \sum_{p=0}^{\infty} \frac{(\xi/2)^{2p}}{p!^2} \Big\{ V(\pi,\pi;p;\sigma) \tag{13}$$

$$- 2\alpha\, V(\pi,\alpha\pi;p;\sigma) + \alpha^2\, V(\alpha\pi,\alpha\pi;p;\sigma) \Big\}$$

$$h(\rho) = \frac{8}{\pi^2\sigma^2} \frac{f_0\, e^{-\rho^2/\sigma^2}}{(1-\alpha^2)^2} \sum_{p=0}^{\infty} \frac{(\xi/2)^{2p}}{p!^2} \frac{\sigma^{2p}\,\sigma^2\,\pi^2}{8} \tag{14}$$

$$\times \Big\{ \sum_{m=0}^{\infty} \frac{(m+p)!\,(-1)^m (\pi\sigma/2)^{2m}}{m!\,(m+1)!} F(-m,-m-1;2;1)$$

$$-2\,\alpha^2 \sum_{m=0}^{\infty} \frac{(m+p)!\,(-1)^m\,(\pi\sigma/2)^{2m}}{m!\,(m+1)!} F(-m,-m-1;2;\alpha^2)$$

$$+ \alpha^4 \sum_{m=0}^{\infty} \frac{(m+p)!\,(-1)^m\,(\pi\alpha\sigma/2)^{2m}}{m!\,(m+1)!} F(-m,-m-1;2;1) \Big\}$$

$$h(\rho) = \frac{f_0\, e^{-\rho^2/\sigma^2}}{(1-\alpha^2)^2} \sum_{p=0}^{\infty} \frac{(\sigma\xi/2)^{2p}}{p!^2} \sum_{m=0}^{\infty} \frac{(m+p)!\,(-1)^m\,(\pi\sigma/2)^{2m}}{m!\,(m+1)!} \tag{15}$$

$$\times \left\{ F(-m,-m-1;2;1) - 2\,\alpha^2\, F(-m,-m-1;2;\alpha^2) + \alpha^{4+2m}\, F(-m,-m-1;2;1) \right\}$$

Uniform convergence over ρ has been assumed.

Interchanging the order of summation,

$$h(\rho) = \frac{f_0 e^{-\rho^2/\sigma^2}}{(1-\alpha^2)^2} \sum_{m=0}^{\infty} \sum_{p=0}^{\infty} \frac{(-1)^m \left(\frac{\pi\sigma}{2}\right)^{2m}}{(m+1)!} \left[\frac{(m+P)!}{m!p!} \frac{[\sigma^2 \ \xi^2/4]^p}{p!}\right] \tag{16}$$

$$x \left\{F(-m,-m-1;2;1) - 2\,\alpha^2 F(-m,-m-1;2;\alpha^2) + \alpha^{4+2m} F(-m,-m-1;2;1)\right\}$$

$$h(\rho) = \frac{f_0 e^{-\rho^2/\sigma^2}}{(1-\alpha^2)^2} \sum_{m=0}^{\infty} \frac{\left(\frac{\pi\sigma}{2}\right)^{2m} (-1)^m}{(m+1)!} \tag{17}$$

$$x \left\{F(-m,-m-1;2;1) - 2\,\alpha^2 F(-m,-m-1;2;\alpha^2) + \alpha^{4+2m} F(-m,-m-1;2;1)\right\}$$

$$x \left\{\sum_{p=0}^{\infty} \frac{\left(\frac{\sigma^2 \ \xi^2}{4}\right)^p}{p!} \left[\frac{(m+p)!}{m!p!}\right]\right\}$$

We pause momentarily to verify

$$\sum_{p=0}^{\infty} \frac{x^p}{p!} \left[\frac{(m+p)!}{m!p!}\right] = e^x L_m(-x) \tag{18}$$

Proof: $\frac{d}{dx} (x^{m+p}) = (m+p)\ x^{m+p-1}$

$$\frac{d^2}{dx^2} (x^{m+p}) = (m+p)\ (m+p-1)\ x^{m+p-2}$$

$$\frac{d^m}{dx^m} (x^{m+p}) = (m+p)\ (m+p-1) \cdots (m+p-m+1)\ x^{m+p-m}$$

$$= \frac{(m+p)!}{p!}\ x^p$$

$$\frac{d^m}{dx^m} x^m e^x = \frac{d^m}{dx^m} \sum_{p=0}^{\infty} \frac{x^{m+p}}{p!}$$

$$= \sum_{p=0}^{\infty} \frac{x^p}{p!} \frac{(m+p)!}{p!}$$

$$\frac{1}{m!}\frac{d^m}{dx^m}\left((-x)^m e^x\right) = \frac{(-1)^m}{m!}\frac{d^m}{dx^m} x^m e^x$$

$$= \frac{(-1)^m}{m!}\sum_{p=0}^{\infty}\frac{x^p}{p!}\left[\frac{(m+p)!}{p!}\right]$$

$$\frac{(-1)^m}{m!}\frac{d^m}{dx^m}\left((-x)^m e^x\right) = \sum_{p=0}^{\infty}\frac{x^p}{p!}\left[\frac{(m+p)!}{m!p!}\right]$$

By definition, the m-th order Laguerre polynomial is

$$L_m(x) \equiv \frac{e^x}{m!}\frac{\partial^m}{\partial x^m}\left(e^{-x} x^m\right)$$

$$L_m(-x) = \frac{e^{-x}}{m!}\frac{\partial m}{\partial(-x)^m}\left(e^x (-x)^m\right)$$

$$e^x L_m(-x) = \frac{(-1)^m}{m!}\frac{\partial^m}{\partial x^m}(-x)^m e^x$$

$$= \sum_{p=0}^{\infty}\frac{x^p}{p!}\left[\frac{(m+p)!}{m!p!}\right]$$

From eqn (18)

$$\sum_{p=0}^{\infty}\frac{\left[\left(\frac{\sigma\xi}{2}\right)^2\right]^p}{p!}\left[\frac{(m+p)!}{m!p!}\right] = e^{\left(\frac{\sigma\xi}{2}\right)^2} L_m\left(-\left(\frac{\sigma\xi}{2}\right)^2\right) \tag{19}$$

From eqn (17)

$$h(\rho) = \frac{f_0\, e^{-\rho^2/\sigma^2}}{(1-\alpha^2)^2}\sum_{M=0}^{\infty}\frac{\left(\frac{\pi\sigma}{2}\right)^{2m}(-1)^m}{(m+1)!} \tag{20}$$

$$\times\left\{F(-m,-m-1;2;1) - 2\,\alpha^2 F(-m,-m-1;2;\alpha^2) + \alpha^{4+2m} F(-m,-m-1;2;1)\right\}$$

$$\times\left[L_m\left(-\left(\frac{\sigma\xi}{2}\right)^2\right) e^{\left(\frac{\sigma\xi}{2}\right)^2}\right]$$

Recall eqn (9)

$$\xi = \frac{2\rho}{\sigma^2}$$

Therefore, from eqns (9) and (20)

$$h(\rho) = \frac{f_0}{(1-\alpha^2)^2} \sum_{m=0}^{\infty} \frac{\left(\frac{\pi\sigma}{2}\right)^{2m}(-1)^m}{(m+1)} L_m\left(-(\rho/\sigma)^2\right) \tag{21}$$

$$\times \left\{F(-m,-m-1;2;1) - 2\,\alpha^2\, F(-m,-m-1;2;\alpha^2) + \alpha^{4+2m}\, F(-m,-m-1;2;1)\right\}$$

F = Hypergeometric function

L_m = Laguerre polynomial, order m

By direct comparison of eqns (1), (2) and (3) in this appendix with eqns (2), (3), and (1) in the main body of this report, one sees that

$$h(\rho) = \langle I(\rho) \rangle \tag{22}$$

when $f_0 = I(0)$.

APPENDIX C

NUMERICAL RESULTS OF JITTER CALCULATIONS

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

		HOLE RADIUS										
		0.00	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
	.00	1.0000	[illegible]	.9877	.9727	.9520	.9261	.8956	.8609	.8228	.7820	.7391
	.05	.9939	[illegible]	.9817	.9665	.9463	.9207	.8905	.8562	.8184	.7780	.7355
R	.10	.9755	.9725	.9641	.9496	.9297	.9049	.8755	.8422	.8056	.7662	.7249
	.15	.9470	.9441	.9360	.9220	.9031	.8795	.8515	.8198	.7849	.7473	.7079
	.20	.9099	.9063	.8983	.8856	.8660	.8459	.8197	.7901	.7574	.7222	.6852
M	.25	.8636	.8612	.8541	.8423	.8261	.8059	.7819	.7546	.7245	.6921	.6579
	.30	.8134	.8112	.8044	.7942	.7796	.7613	.7395	.7149	.6875	.6582	.6271
	.35	.7603	.7584	.7526	.7432	.7302	.7133	.6946	.6726	.6482	.6219	.5940
S	.40	.7062	.7045	.6995	.6912	.6796	.6655	.6485	.6291	.6076	.5844	.5596
	.45	.6526	.6514	.6470	.6338	.6299	.6175	.6027	.5858	.5670	.5466	.5250
	.50	.6013	.6001	.5963	.5902	.5817	.5709	.5582	.5436	.5273	.5095	.4907
	.55	.5520	.5516	.5484	.5431	.5358	.5266	.5157	.5032	.4892	.4739	.4576
	.60	.5072	.5063	.5036	.4991	.4929	.4851	.4758	.4651	.4531	.4400	.4260
J	.65	.4654	.4640	.4623	.4585	.4533	.4467	.4387	.4296	.4194	.4082	.3961
	.70	.4272	.4265	.4246	.4214	.4169	.4113	.4046	.3969	.3881	.3785	.3682
	.75	.3925	.3919	.3903	.3876	.3838	.3790	.3733	.3667	.3593	.3511	.3423
I	.80	.3611	.3606	.3592	.3569	.3537	.3497	.3448	.3392	.3329	.3259	.3183
	.85	.3326	.3324	.3312	.3293	.3265	.3231	.3190	.3142	.3087	.3028	.2963
	.90	.3073	.3069	.3059	.3043	.3020	.2990	.2955	.2914	.2867	.2816	.2760
T	.95	.2843	.2840	.2831	.2817	.2797	.2772	.2742	.2707	.2667	.2623	.2575
	1.00	.2636	.2633	.2625	.2614	.2597	.2575	.2549	.2519	.2484	.2446	.2405
	1.05	.2449	.2447	.2440	.2430	.2415	.2396	.2374	.2348	.2318	.2285	.2249
T	1.10	.2279	.2276	.2272	.2265	.2251	.2234	.2215	.2192	.2167	.2138	.2106
	1.15	.2126	.2125	.2120	.2112	.2101	.2087	.2071	.2051	.2028	.2003	.1976
	1.20	.1987	.1986	.1982	.1975	.1965	.1953	.1938	.1921	.1902	.1880	.1856
E	1.25	.1861	.1860	.1856	.1851	.1842	.1831	.1813	.1803	.1786	.1767	.1746
	1.30	.1746	.1745	.1742	.1736	.1729	.1720	.1703	.1695	.1680	.1663	.1644
	1.35	.1641	.1640	.1637	.1632	.1626	.1617	.1607	.1596	.1582	.1567	.1551
R	1.40	.1544	.1543	.1541	.1537	.1531	.1524	.1515	.1505	.1493	.1480	.1465
	1.45	.1456	.1455	.1453	.1449	.1444	.1438	.1430	.1421	.1410	.1399	.1386
	1.50	.1375	.1374	.1372	.1369	.1365	.1359	.1352	.1344	.1334	.1324	.1312

A. GIVEN

1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN

THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*,AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO A PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .0000   T = 1-ALPHA**2 = 1.0000

AVERAGE IRRADIANCE OVER A CIRCULAR HOLE

HOLE RADIUS

		0.00	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50
	1.50	.1375	.1374	.1372	.1369	.1365	.1359	.1352	.1344	.1334	.1324	.1312
	1.55	.1300	.1300	.1298	.1295	.1291	.1286	.1280	.1272	.1264	.1255	.1244
R	1.60	.1231	.1231	.1229	.1227	.1223	.1213	.1213	.1206	.1199	.1190	.1181
	1.65	.1168	.1167	.1166	.1163	.1160	.1156	.1151	.1145	.1139	.1131	.1123
	1.70	.1109	.1109	.1107	.1103	.1102	.1098	.1094	.1089	.1082	.1076	.1068
M	1.75	.1054	.1054	.1053	.1051	.1048	.1045	.1041	.1036	.1030	.1024	.1017
	1.80	.1003	.1003	.1002	.1000	.0998	.0995	.0991	.0987	.0982	.0976	.0970
	1.85	.0956	.0956	.0955	.0953	.0951	.0948	.0945	.0941	.0937	.0931	.0926
S	1.90	.0912	.0912	.0911	.0909	.0907	.0905	.0902	.0898	.0894	.0890	.0885
	1.95	.0871	.0871	.0870	.0869	.0867	.0865	.0862	.0858	.0855	.0851	.0846
	2.00	.0832	.0832	.0831	.0830	.0829	.0827	.0824	.0821	.0818	.0814	.0810
	2.05	.0796	.0796	.0796	.0795	.0793	.0791	.0789	.0786	.0783	.0780	.0776
	2.10	.0763	.0763	.0762	.0761	.0760	.0758	.0756	.0753	.0750	.0747	.0744
J	2.15	.0731	.0731	.0730	.0730	.0728	.0727	.0725	.0722	.0720	.0717	.0714
	2.20	.0701	.0701	.0701	.0700	.0699	.0697	.0696	.0693	.0691	.0688	.0685
	2.25	.0673	.0673	.0673	.0672	.0671	.0670	.0669	.0666	.0664	.0661	.0659
I	2.30	.0647	.0647	.0647	.0646	.0645	.0644	.0642	.0640	.0639	.0636	.0633
	2.35	.0622	.0622	.0622	.0621	.0620	.0619	.0618	.0616	.0614	.0612	.0610
	2.40	.0599	.0599	.0598	.0598	.0597	.0596	.0595	.0593	.0591	.0589	.0587
T	2.45	.0577	.0577	.0576	.0576	.0575	.0574	.0573	.0571	.0570	.0568	.0566
	2.50	.0556	.0556	.0555	.0555	.0554	.0553	.0552	.0551	.0549	.0547	.0546
	2.55	.0536	.0536	.0535	.0535	.0534	.0533	.0532	.0531	.0530	.0528	.0526
T	2.60	.0517	.0517	.0517	.0516	.0516	.0515	.0514	.0513	.0511	.0510	.0508
	2.65	.0499	.0499	.0499	.0499	.0498	.0497	.0496	.0495	.0494	.0493	.0491
	2.70	.0482	.0482	.0482	.0482	.0481	.0480	.0480	.0479	.0477	.0476	.0475
E	2.75	.0466	.0466	.0466	.0466	.0465	.0464	.0464	.0463	.0462	.0460	.0459
	2.80	.0451	.0451	.0451	.0451	.0450	.0449	.0448	.0448	.0447	.0445	.0444
	2.85	.0436	.0436	.0436	.0436	.0435	.0435	.0434	.0433	.0432	.0431	.0430
R	2.90	.0422	.0422	.0422	.0422	.0421	.0421	.0420	.0420	.0419	.0418	.0417
	2.95	.0409	.0409	.0409	.0409	.0408	.0408	.0407	.0407	.0406	.0405	.0404
	3.00	.0397	.0397	.0396	.0396	.0396	.0395	.0395	.0394	.0393	.0392	.0391

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO E-M PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
*. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
R = S*D/(F*WL)
8. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .0000 T = 1-ALPHA**2 = 1.0000

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

HOLE RADIUS

		.50	.55	.60	.65	.70	.75	.80	.85	.90	.95	1.00
	0.00	.7331	.6949	.6501	.6053	.5612	.5132	.4769	.4377	.4008	.3665	.3349
	.05	.7355	.6917	.6473	.6029	.5591	.5165	.4756	.4366	.4000	.3659	.3345
R	.10	.7249	.6825	.6371	.5950	.5531	.5113	.4715	.4334	.3975	.3640	.3332
	.15	.7079	.6672	.6255	.5843	.5434	.5034	.4649	.4280	.3933	.3609	.3309
	.20	.6852	.6469	.6033	.5609	.5302	.4923	.4557	.4207	.3876	.3565	.3277
M	.25	.6579	.6225	.5864	.5501	.5141	.4787	.4444	.4115	.3803	.3509	.3234
	.30	.6271	.5943	.5619	.5286	.4953	.4623	.4312	.4007	.3715	.3440	.3182
	.35	.5940	.5650	.5354	.5052	.4751	.4455	.4165	.3884	.3615	.3360	.3119
S	.40	.5596	.5339	.5073	.4804	.4535	.4267	.4005	.3750	.3504	.3269	.3046
	.45	.5250	.5023	.4789	.4550	.4310	.4071	.3836	.3606	.3383	.3169	.2964
	.50	.4907	.4719	.4504	.4235	.4083	.3871	.3661	.3455	.3255	.3061	.2875
	.55	.4576	.4404	.4226	.4043	.3857	.3670	.3485	.3301	.3122	.2947	.2779
	.60	.4260	.4111	.3957	.3797	.3635	.3472	.3309	.3146	.2986	.2830	.2678
J	.65	.3961	.3833	.3700	.3562	.3421	.3279	.3134	.2991	.2849	.2710	.2574
	.70	.3682	.3572	.3457	.3338	.3215	.3090	.2965	.2839	.2713	.2590	.2468
	.75	.3423	.3325	.3223	.3125	.3020	.2911	.2801	.2690	.2560	.2470	.2362
I	.80	.3133	.3102	.3017	.2923	.2835	.2741	.2645	.2547	.2450	.2353	.2256
	.85	.2963	.2893	.2813	.2742	.2662	.2590	.2496	.2410	.2325	.2238	.2153
	.90	.2760	.2733	.2636	.2570	.2500	.2423	.2355	.2280	.2204	.2128	.2052
T	.95	.2575	.2523	.2468	.2410	.2349	.2286	.2222	.2156	.2089	.2022	.1954
	1.00	.2405	.2360	.2312	.2261	.2208	.2154	.2097	.2039	.1980	.1921	.1861
	1.05	.2249	.2210	.2168	.2124	.2078	.2030	.1980	.1929	.1877	.1824	.1771
T	1.10	.2106	.2072	.2036	.1997	.1957	.1915	.1871	.1826	.1780	.1733	.1685
	1.15	.1976	.1946	.1914	.1880	.1844	.1807	.1769	.1729	.1688	.1646	.1604
	1.20	.1856	.1830	.1802	.1772	.1740	.1709	.1673	.1638	.1602	.1565	.1527
E	1.25	.1746	.1723	.1698	.1672	.1644	.1615	.1584	.1553	.1521	.1488	.1454
	1.30	.1644	.1624	.1602	.1573	.1554	.1528	.1502	.1474	.1445	.1415	.1385
	1.35	.1551	.1533	.1514	.1493	.1471	.1448	.1424	.1399	.1373	.1347	.1320
R	1.40	.1465	.1449	.1432	.1413	.1394	.1373	.1352	.1330	.1307	.1283	.1258
	1.45	.1386	.1371	.1356	.1340	.1322	.1304	.1285	.1265	.1244	.1223	.1201
	1.50	.1312	.1299	.1286	.1271	.1256	.1239	.1222	.1204	.1185	.1166	.1146

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO E-M PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .0000 T = 1-ALPHA**2 = 1.0000

AVERAGE IRRADIANCE OVER A CIRCULAR HOLE

RMS JITTER

	HOLE RADIUS .50	.55	.60	.65	.70	.75	.80	.85	.90	.95	1.00
1.50	.1312	[illegible]	[illegible]	.1271	.1256	.1239	.1222	.1204	.1185	.1166	.1146
1.55	.1244	[illegible]	.1220	.1207	.1193	.1179	.1163	.1147	.1130	.1113	.1095
1.60	.1151	.1171	.1160	.1146	.1135	.1122	.1108	.1094	.1079	.1063	.1046
1.65	.1123	.1113	.1106	.1093	.1081	.1069	.1057	.1044	.1030	.1015	.1001
1.70	.1066	.1058	.1051	.1041	.1031	.1020	.1009	.0997	.0984	.0971	.0958
1.75	.1017	.1010	.1002	.0993	.0984	.0974	.0963	.0953	.0941	.0929	.0917
1.80	.0970	.0963	.0956	.0948	.0940	.0931	.0921	.0911	.0901	.0890	.0879
1.85	.0926	.0920	.0913	.0906	.0898	.0890	.0881	.0872	.0863	.0853	.0843
1.90	.0885	.0879	.0873	.0866	.0859	.0852	.0844	.0836	.0827	.0818	.0809
1.95	.0846	.0841	.0835	.0829	.0823	.0816	.0809	.0801	.0793	.0785	.0777
2.00	.0810	.0805	.0800	.0794	.0789	.0782	.0776	.0769	.0762	.0754	.0746
2.05	.0776	.0771	.0767	.0762	.0756	.0751	.0745	.0738	.0732	.0725	.0717
2.10	.0744	.0740	.0735	.0731	.0726	.0721	.0715	.0709	.0703	.0697	.0690
2.15	.0714	.0710	.0706	.0702	.0697	.0692	.0687	.0682	.0676	.0670	.0664
2.20	.0685	.0682	.0678	.0674	.0670	.0666	.0661	.0656	.0651	.0646	.0640
2.25	.0659	.0656	.0652	.0649	.0645	.0641	.0636	.0632	.0627	.0622	.0617
2.30	.0633	.0631	.0627	.0624	.0621	.0617	.0613	.0609	.0604	.0599	.0595
2.35	.0610	.0607	.0604	.0601	.0598	.0594	.0591	.0587	.0582	.0578	.0574
2.40	.0587	.0585	.0582	.0579	.0576	.0573	.0569	.0566	.0562	.0558	.0554
2.45	.0566	.0564	.0561	.0558	.0556	.0553	.0549	.0546	.0542	.0539	.0535
2.50	.0546	.0544	.0541	.0539	.0536	.0533	.0530	.0527	.0524	.0520	.0517
2.55	.0526	.0525	.0522	.0520	.0518	.0515	.0512	.0509	.0506	.0503	.0500
2.60	.0508	.0507	.0505	.0502	.0500	.0498	.0495	.0492	.0490	.0487	.0483
2.65	.0491	.0489	.0488	.0486	.0483	.0481	.0479	.0476	.0474	.0471	.0468
2.70	.0475	.0473	.0471	.0470	.0468	.0465	.0463	.0461	.0458	.0456	.0453
2.75	.0459	.0458	.0456	.0454	.0452	.0450	.0448	.0446	.0444	.0441	.0439
2.80	.0444	.0443	.0441	.0440	.0438	.0436	.0434	.0432	.0430	.0428	.0425
2.85	.0430	.0429	.0427	.0426	.0424	.0423	.0421	.0419	.0417	.0415	.0412
2.90	.0417	.0415	.0414	.0413	.0411	.0410	.0408	.0406	.0404	.0402	.0400
2.95	.0404	.0403	.0401	.0400	.0399	.0397	.0395	.0394	.0392	.0390	.0388
3.00	.0391	.0390	.0389	.0388	.0387	.0385	.0384	.0382	.0380	.0379	.0377

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO 1-M PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .0000 T = 1-ALPHA**2 = 1.0000

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

HOLE RADIUS

		1.00	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
	0.00	.3349	.3361	.2800	.2566	.2358	.2173	.2010	.1867	.1740	.1629	.1531
	.05	.3345	.3355	.2743	.2566	.2358	.2173	.2011	.1867	.1741	.1630	.1531
R	.10	.3332	.3343	.2743	.2563	.2357	.2174	.2012	.1869	.1743	.1631	.1533
	.15	.3309	.3334	.2733	.2557	.2355	.2174	.2013	.1871	.1745	.1633	.1534
	.20	.3277	.3011	.2763	.2549	.2350	.2173	.2014	.1873	.1747	.1636	.1536
M	.25	.3234	.2931	.2743	.2535	.2343	.2169	.2014	.1874	.1749	.1638	.1538
	.30	.3162	.2441	.2723	.2516	.2331	.2152	.2010	.1873	.1750	.1639	.1539
	.35	.3119	.2433	.2634	.2491	.2313	.2151	.2006	.1870	.1748	.1639	.1539
S	.40	.3046	.2536	.2643	.2451	.2289	.2134	.1992	.1862	.1744	.1636	.1538
	.45	.2904	.2771	.2583	.2413	.2259	.2112	.1976	.1851	.1736	.1631	.1534
	.50	.2375	.2637	.2523	.2371	.2222	.2083	.1954	.1834	.1724	.1622	.1528
	.55	.2779	.2517	.2463	.2317	.2176	.2043	.1927	.1813	.1707	.1609	.1518
	.60	.2676	.2531	.2391	.2257	.2129	.2008	.1894	.1787	.1686	.1592	.1505
J	.65	.2574	.2442	.2314	.2191	.2074	.1962	.1856	.1756	.1661	.1572	.1488
	.70	.2466	.2349	.2234	.2122	.2015	.1912	.1814	.1720	.1631	.1547	.1468
	.75	.2362	.2255	.2151	.2050	.1953	.1858	.1768	.1681	.1598	.1519	.1445
I	.80	.2256	.2161	.2063	.1977	.1888	.1802	.1719	.1638	.1562	.1488	.1418
	.85	.2153	.2063	.1954	.1902	.1821	.1743	.1667	.1593	.1522	.1454	.1389
	.90	.2052	.1970	.1901	.1827	.1755	.1683	.1614	.1546	.1491	.1418	.1357
T	.95	.1954	.1887	.1823	.1753	.1686	.1623	.1560	.1498	.1433	.1380	.1323
	1.00	.1861	.1800	.1740	.1651	.1621	.1563	.1505	.1449	.1394	.1340	.1288
	1.05	.1771	.1717	.1663	.1610	.1556	.1503	.1451	.1400	.1350	.1300	.1252
T	1.10	.1665	.1637	.1593	.1541	.1493	.1445	.1398	.1351	.1305	.1260	.1215
	1.15	.1604	.1561	.1513	.1474	.1431	.1388	.1345	.1302	.1260	.1219	.1178
	1.20	.1527	.1486	.1450	.1411	.1371	.1332	.1293	.1255	.1216	.1178	.1141
E	1.25	.1454	.1419	.1385	.1349	.1314	.1279	.1243	.1208	.1173	.1138	.1104
	1.30	.1385	.1354	.1323	.1291	.1259	.1227	.1135	.1163	.1131	.1099	.1068
	1.35	.1320	.1292	.1264	.1235	.1206	.1177	.1148	.1119	.1090	.1061	.1032
R	1.40	.1258	.1233	.1205	.1182	.1156	.1130	.1103	.1076	.1050	.1023	.0997
	1.45	.1201	.1176	.1155	.1132	.1108	.1084	.1060	.1035	.1011	.0987	.0962
	1.50	.1146	.1125	.1105	.1084	.1062	.1040	.1018	.0996	.0974	.0951	.0929

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO A PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2))/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .0000 T = 1-ALPHA**2 = 1.0000

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

HOLE RADIUS

		1.00	1.05	1.10	1.15	1.20	1.25	1.30	1.35	1.40	1.45	1.50
	1.50	.1146	.1126	.1105	.1084	.1062	.1040	.1018	.0996	.0974	.0951	.0929
	1.55	.1095	.1076	.1057	.1033	.1018	.0999	.0979	.0956	.0938	.0917	.0897
R	1.60	.1046	.1030	.1012	.0995	.0977	.0959	.0941	.0922	.0903	.0885	.0866
	1.65	.1001	.0985	.0970	.0954	.0938	.0921	.0904	.0887	.0870	.0853	.0836
	1.70	.0958	.0944	.0930	.0915	.0900	.0885	.0870	.0854	.0839	.0823	.0807
M	1.75	.0917	.0905	.0892	.0878	.0865	.0851	.0837	.0823	.0808	.0793	.0779
	1.80	.0879	.0867	.0856	.0843	.0831	.0818	.0805	.0792	.0779	.0766	.0752
	1.85	.0843	.0832	.0822	.0810	.0799	.0787	.0775	.0763	.0751	.0739	.0726
S	1.90	.0809	.0799	.0789	.0779	.0769	.0758	.0747	.0736	.0724	.0713	.0701
	1.95	.0777	.0768	.0759	.0749	.0740	.0730	.0720	.0709	.0699	.0688	.0678
	2.00	.0746	.0739	.0730	.0721	.0712	.0703	.0694	.0684	.0675	.0665	.0655
	2.05	.0717	.0710	.0702	.0694	.0686	.0678	.0669	.0660	.0651	.0642	.0633
	2.10	.0690	.0683	.0676	.0669	.0661	.0653	.0645	.0637	.0629	.0621	.0612
J	2.15	.0664	.0658	.0651	.0645	.0638	.0630	.0623	.0616	.0608	.0600	.0592
	2.20	.0640	.0634	.0628	.0622	.0615	.0608	.0602	.0595	.0588	.0580	.0573
	2.25	.0617	.0611	.0606	.0600	.0594	.0588	.0581	.0575	.0568	.0561	.0555
I	2.30	.0595	.0590	.0584	.0579	.0573	.0568	.0562	.0556	.0550	.0543	.0537
	2.35	.0574	.0569	.0564	.0559	.0554	.0549	.0543	.0538	.0532	.0526	.0520
	2.40	.0554	.0549	.0545	.0540	.0535	.0530	.0525	.0520	.0515	.0509	.0504
T	2.45	.0535	.0531	.0527	.0522	.0518	.0513	.0508	.0504	.0499	.0493	.0488
	2.50	.0517	.0513	.0509	.0505	.0501	.0497	.0492	.0488	.0483	.0478	.0473
	2.55	.0500	.0496	.0493	.0489	.0485	.0481	.0477	.0472	.0468	.0464	.0459
T	2.60	.0483	.0480	.0477	.0473	.0470	.0466	.0462	.0458	.0454	.0450	.0445
	2.65	.0468	.0465	.0462	.0458	.0455	.0451	.0448	.0444	.0440	.0436	.0432
	2.70	.0453	.0450	.0447	.0444	.0441	.0438	.0434	.0431	.0427	.0423	.0420
E	2.75	.0439	.0436	.0433	.0430	.0427	.0424	.0421	.0418	.0415	.0411	.0408
	2.80	.0425	.0423	.0420	.0417	.0415	.0412	.0409	.0406	.0402	.0399	.0396
	2.85	.0412	.0410	.0407	.0405	.0402	.0400	.0397	.0394	.0391	.0388	.0385
R	2.90	.0400	.0398	.0395	.0393	.0391	.0388	.0385	.0383	.0380	.0377	.0374
	2.95	.0388	.0386	.0384	.0382	.0379	.0377	.0374	.0372	.0369	.0366	.0364
	3.00	.0377	.0375	.0373	.0371	.0368	.0366	.0364	.0361	.0359	.0356	.0354

A. GIVEN

1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN

THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO E-M PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .0000 T = 1-ALPHA**2 = 1.0000

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

		HOLE RADIUS										
		1.50	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
	0.00	.1531	.1444	.1365	.1294	.1229	.1169	.1113	.1060	.1010	.0963	.0918
	.05	.1531	.1444	.1365	.1294	.1229	.1169	.1113	.1060	.1010	.0963	.0918
R	.10	.1533	.1445	.1365	.1294	.1228	.1168	.1112	.1059	.1009	.0962	.0917
	.15	.1534	.1446	.1366	.1294	.1228	.1167	.1110	.1058	.1008	.0961	.0916
	.20	.1536	.1447	.1367	.1294	.1227	.1166	.1109	.1056	.1006	.0959	.0915
M	.25	.1538	.1448	.1367	.1294	.1226	.1165	.1107	.1054	.1004	.0957	.0913
	.30	.1539	.1449	.1368	.1293	.1226	.1163	.1106	.1052	.1002	.0955	.0912
	.35	.1539	.1448	.1368	.1293	.1225	.1162	.1104	.1050	.1000	.0954	.0910
S	.40	.1538	.1448	.1367	.1292	.1224	.1161	.1102	.1049	.0999	.0952	.0908
	.45	.1534	.1446	.1365	.1290	.1222	.1159	.1101	.1047	.0997	.0950	.0906
	.50	.1528	.1441	.1361	.1288	.1220	.1157	.1099	.1045	.0995	.0948	.0905
	.55	.1518	.1434	.1355	.1283	.1216	.1154	.1096	.1043	.0993	.0947	.0903
	.60	.1505	.1423	.1347	.1277	.1211	.1150	.1093	.1040	.0991	.0944	.0901
J	.65	.1488	.1410	.1337	.1268	.1204	.1144	.1088	.1036	.0987	.0942	.0899
	.70	.1468	.1393	.1323	.1257	.1195	.1137	.1082	.1031	.0983	.0938	.0896
	.75	.1445	.1374	.1307	.1243	.1184	.1128	.1075	.1025	.0978	.0934	.0893
I	.80	.1418	.1351	.1288	.1227	.1170	.1116	.1065	.1017	.0972	.0929	.0888
	.85	.1389	.1326	.1265	.1209	.1155	.1103	.1054	.1008	.0964	.0922	.0883
	.90	.1357	.1295	.1242	.1188	.1137	.1088	.1041	.0996	.0954	.0914	.0876
T	.95	.1323	.1264	.1216	.1165	.1117	.1071	.1026	.0984	.0943	.0905	.0868
	1.00	.1288	.1233	.1189	.1141	.1096	.1052	.1010	.0970	.0931	.0894	.0858
	1.05	.1252	.1205	.1160	.1116	.1073	.1032	.0992	.0954	.0917	.0882	.0848
T	1.10	.1215	.1172	.1130	.1089	.1049	.1010	.0973	.0937	.0902	.0869	.0836
	1.15	.1178	.1138	.1099	.1061	.1024	.0988	.0953	.0919	.0886	.0854	.0824
	1.20	.1141	.1104	.1068	.1033	.0999	.0965	.0932	.0900	.0869	.0839	.0810
E	1.25	.1104	.1070	.1037	.1004	.0972	.0941	.0910	.0880	.0851	.0823	.0796
	1.30	.1068	.1036	.1006	.0976	.0946	.0917	.0888	.0860	.0833	.0806	.0780
	1.35	.1032	.1003	.0975	.0947	.0919	.0892	.0866	.0839	.0814	.0789	.0765
R	1.40	.0997	.0970	.0944	.0918	.0893	.0868	.0843	.0819	.0795	.0771	.0748
	1.45	.0962	.0935	.0914	.0890	.0867	.0843	.0820	.0798	.0775	.0753	.0732
	1.50	.0929	.0907	.0885	.0863	.0841	.0819	.0798	.0777	.0756	.0735	.0715

A. GIVEN

1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN

THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO E-M PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .0000 T = 1-ALPHA**2 = 1.0000

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

HOLE RADIUS

RMS JITTER

	1.50	1.55	1.60	1.65	1.70	1.75	1.80	1.85	1.90	1.95	2.00
1.50	.0929	.0907	.0885	.0863	.0841	.0819	.0798	.0777	.0756	.0735	.0715
1.55	.0897	.0876	.0856	.0836	.0815	.0795	.0775	.0756	.0736	.0717	.0698
1.60	.0866	.0847	.0828	.0809	.0790	.0772	.0753	.0735	.0717	.0699	.0681
1.65	.0836	.0818	.0801	.0783	.0766	.0749	.0732	.0715	.0698	.0681	.0664
1.70	.0807	.0791	.0775	.0758	.0742	.0726	.0710	.0694	.0679	.0663	.0648
1.75	.0779	.0764	.0749	.0734	.0719	.0704	.0690	.0675	.0660	.0645	.0631
1.80	.0752	.0738	.0725	.0711	.0697	.0683	.0669	.0655	.0642	.0628	.0615
1.85	.0726	.0714	.0701	.0688	.0675	.0662	.0649	.0637	.0624	.0611	.0598
1.90	.0701	.0688	.0678	.0666	.0654	.0642	.0630	.0618	.0606	.0594	.0583
1.95	.0678	.0667	.0656	.0645	.0634	.0623	.0612	.0600	.0589	.0578	.0567
2.00	.0655	.0645	.0635	.0625	.0614	.0604	.0594	.0583	.0573	.0562	.0552
2.05	.0633	.0624	.0614	.0605	.0595	.0586	.0576	.0566	.0557	.0547	.0537
2.10	.0612	.0604	.0595	.0586	.0577	.0568	.0559	.0550	.0541	.0532	.0523
2.15	.0592	.0584	.0576	.0568	.0559	.0551	.0543	.0534	.0526	.0517	.0509
2.20	.0573	.0565	.0558	.0550	.0542	.0535	.0527	.0519	.0511	.0503	.0495
2.25	.0555	.0548	.0541	.0533	.0526	.0519	.0511	.0504	.0497	.0489	.0482
2.30	.0537	.0530	.0524	.0517	.0510	.0504	.0497	.0490	.0483	.0476	.0469
2.35	.0520	.0514	.0508	.0502	.0495	.0489	.0482	.0476	.0469	.0463	.0456
2.40	.0504	.0499	.0492	.0487	.0481	.0475	.0469	.0462	.0456	.0450	.0444
2.45	.0488	.0483	.0478	.0472	.0467	.0461	.0455	.0450	.0444	.0438	.0432
2.50	.0473	.0468	.0463	.0458	.0453	.0448	.0443	.0437	.0432	.0426	.0421
2.55	.0459	.0454	.0450	.0445	.0440	.0435	.0430	.0425	.0420	.0415	.0410
2.60	.0445	.0441	.0437	.0432	.0428	.0423	.0418	.0413	.0409	.0404	.0399
2.65	.0432	.0428	.0424	.0420	.0416	.0411	.0407	.0402	.0398	.0393	.0389
2.70	.0420	.0416	.0412	.0408	.0404	.0400	.0396	.0391	.0387	.0383	.0379
2.75	.0408	.0404	.0400	.0397	.0393	.0389	.0385	.0381	.0377	.0373	.0369
2.80	.0396	.0393	.0389	.0386	.0382	.0378	.0375	.0371	.0367	.0363	.0359
2.85	.0385	.0382	.0378	.0375	.0372	.0368	.0365	.0361	.0358	.0354	.0350
2.90	.0374	.0371	.0368	.0365	.0362	.0358	.0355	.0352	.0348	.0345	.0342
2.95	.0364	.0361	.0358	.0355	.0352	.0349	.0346	.0343	.0340	.0336	.0333
3.00	.0354	.0351	.0348	.0346	.0343	.0340	.0337	.0334	.0331	.0328	.0325

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS "HOLE RADIUS", AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFER PATTERN DUE TO A PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .0000   T = 1-ALPHA**2 = 1.0000

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

HOLE RADIUS

RMS JITTER	0.00	.20	.40	.60	.80	1.00	1.20	1.40	1.60	1.80	2.00
0.00	1.0000	.9320	.8223	.6501	.4769	.3343	.2358	.1740	.1365	.1113	.0918
.20	.9389	.8630	.7574	.6380	.4557	.3277	.2350	.1747	.1367	.1109	.0915
.40	.7062	.6733	.6076	.5073	.4005	.3046	.2233	.1744	.1367	.1102	.0908
.60	.5072	.4329	.4531	.3957	.3308	.2679	.2123	.1686	.1347	.1093	.0901
.80	.3511	.3537	.3329	.3017	.2645	.2256	.1988	.1562	.1288	.1065	.0888
1.00	.2636	.2597	.2464	.2312	.2097	.1861	.1621	.1394	.1189	.1010	.0858
1.20	.1967	.1965	.1902	.1802	.1673	.1527	.1371	.1215	.1058	.0932	.0810
1.40	.1544	.1531	.1493	.1432	.1352	.1255	.1156	.1050	.0944	.0843	.0748
1.60	.1231	.1223	.1199	.1160	.1108	.1046	.0977	.0903	.0828	.0753	.0681
1.80	.1003	.0998	.0982	.0956	.0921	.0879	.0831	.0779	.0725	.0669	.0615
2.00	.0832	.0829	.0818	.0800	.0776	.0746	.0712	.0675	.0635	.0594	.0552
2.20	.0701	.0699	.0691	.0678	.0661	.0640	.0615	.0589	.0559	.0527	.0495
2.40	.0599	.0597	.0591	.0582	.0569	.0554	.0535	.0515	.0492	.0469	.0444
2.60	.0517	.0516	.0511	.0505	.0495	.0483	.0470	.0454	.0437	.0418	.0399
2.80	.0451	.0450	.0447	.0441	.0434	.0425	.0415	.0402	.0389	.0375	.0359
3.00	.0397	.0396	.0393	.0389	.0364	.0377	.0368	.0359	.0348	.0337	.0325

RMS JITTER	2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00
0.00	.0918	.0762	.0641	.0550	.0480	.0421	.0371	.0329	.0294	.0266	.0241
.20	.0915	.0761	.0642	.0551	.0480	.0421	.0371	.0329	.0295	.0266	.0241
.40	.0908	.0759	.0642	.0551	.0479	.0420	.0371	.0329	.0295	.0266	.0240
.60	.0901	.0755	.0641	.0551	.0478	.0419	.0370	.0329	.0295	.0265	.0240
.80	.0888	.0744	.0637	.0545	.0477	.0418	.0369	.0329	.0294	.0265	.0240
1.00	.0858	.0733	.0623	.0544	.0474	.0416	.0368	.0328	.0294	.0265	.0240
1.20	.0810	.0704	.0612	.0534	.0469	.0413	.0366	.0327	.0293	.0264	.0239
1.40	.0748	.0662	.0585	.0517	.0456	.0407	.0362	.0324	.0292	.0263	.0239
1.60	.0681	.0613	.0553	.0493	.0442	.0396	.0355	.0320	.0289	.0261	.0238
1.80	.0615	.0562	.0511	.0464	.0421	.0381	.0345	.0313	.0284	.0258	.0235
2.00	.0552	.0511	.0471	.0432	.0396	.0362	.0331	.0303	.0277	.0253	.0232
2.20	.0495	.0463	.0431	.0400	.0370	.0342	.0315	.0290	.0267	.0246	.0227
2.40	.0444	.0419	.0393	.0365	.0344	.0320	.0298	.0276	.0256	.0238	.0220
2.60	.0399	.0379	.0359	.0339	.0319	.0299	.0280	.0262	.0244	.0228	.0212
2.80	.0359	.0344	.0327	.0311	.0294	.0278	.0262	.0247	.0232	.0217	.0204
3.00	.0325	.0312	.0299	.0285	.0272	.0258	.0245	.0232	.0219	.0207	.0195

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO I-A PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA =0.0000 T = 1-ALPHA**2 = 1.0000

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

HOLE RADIUS

RMS JITTER	0.00	.20	.40	.60	.80	1.00	1.20	1.40	1.60	1.80	2.00
0.00	1.0000	.9313	.8212	.6471	.4731	.3311	.2327	.1720	.1356	.1112	.0922
.20	.9360	.8997	.7553	.6649	.4520	.3241	.2321	.1729	.1358	.1108	.0917
.40	.7836	.6775	.6047	.5341	.3971	.3016	.2265	.1728	.1358	.1100	.0909
.60	.5041	.4999	.4500	.3927	.3260	.2654	.2110	.1674	.1340	.1090	.0901
.80	.3563	.3510	.3322	.2992	.2623	.2238	.1874	.1552	.1281	.1062	.0887
1.00	.2615	.2576	.2465	.2294	.2081	.1843	.1611	.1386	.1183	.1006	.0857
1.20	.1973	.1951	.1869	.1769	.1662	.1517	.1364	.1210	.1064	.0929	.0808
1.40	.1535	.1522	.1464	.1424	.1345	.1252	.1150	.1045	.0941	.0840	.0747
1.60	.1225	.1217	.1193	.1154	.1103	.1042	.0973	.0900	.0825	.0751	.0680
1.80	.0999	.0994	.0978	.0952	.0918	.0876	.0828	.0777	.0723	.0668	.0613
2.00	.0830	.0826	.0815	.0797	.0773	.0744	.0710	.0673	.0633	.0592	.0551
2.20	.0699	.0697	.0689	.0677	.0659	.0638	.0614	.0586	.0557	.0526	.0494
2.40	.0597	.0596	.0590	.0581	.0568	.0553	.0534	.0514	.0492	.0468	.0443
2.60	.0516	.0515	.0511	.0504	.0494	.0483	.0469	.0453	.0436	.0418	.0399
2.80	.0450	.0449	.0446	.0441	.0434	.0425	.0414	.0402	.0389	.0374	.0359
3.00	.0396	.0395	.0393	.0389	.0383	.0376	.0368	.0359	.0348	.0337	.0325

RMS JITTER	2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00
0.00	.0922	.0766	.0644	.0552	.0480	.0420	.0370	.0328	.0294	.0267	.0242
.20	.0917	.0765	.0645	.0552	.0479	.0420	.0370	.0328	.0294	.0266	.0242
.40	.0909	.0761	.0645	.0552	.0479	.0419	.0370	.0329	.0295	.0266	.0241
.60	.0901	.0756	.0642	.0551	.0478	.0419	.0370	.0329	.0295	.0266	.0241
.80	.0887	.0745	.0637	.0549	.0477	.0418	.0369	.0329	.0295	.0266	.0241
1.00	.0857	.0732	.0629	.0544	.0474	.0416	.0368	.0328	.0294	.0265	.0240
1.20	.0808	.0702	.0611	.0534	.0468	.0413	.0367	.0327	.0293	.0265	.0240
1.40	.0747	.0661	.0584	.0517	.0456	.0407	.0363	.0325	.0292	.0264	.0239
1.60	.0680	.0612	.0553	.0493	.0441	.0396	.0355	.0320	.0289	.0262	.0238
1.80	.0613	.0561	.0511	.0464	.0420	.0381	.0345	.0313	.0284	.0258	.0236
2.00	.0551	.0510	.0470	.0432	.0396	.0362	.0331	.0303	.0277	.0253	.0232
2.20	.0494	.0462	.0431	.0400	.0370	.0342	.0315	.0290	.0267	.0246	.0227
2.40	.0443	.0418	.0393	.0368	.0344	.0320	.0298	.0277	.0257	.0238	.0220
2.60	.0399	.0379	.0359	.0339	.0319	.0299	.0280	.0262	.0245	.0228	.0213
2.80	.0359	.0343	.0327	.0311	.0295	.0278	.0262	.0247	.0232	.0218	.0204
3.00	.0325	.0312	.0299	.0286	.0272	.0259	.0245	.0232	.0219	.0207	.0195

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS "HOLE RADIUS", AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO A PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .1000   T = 1-ALPHA**2 = .9900

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

HOLE RADIUS

		.00	.20	.40	.60	.80	1.00	1.20	1.40	1.60	1.80	2.00
	0.00	1.0000	.9531	.8162	.6383	.4620	.3200	.2239	.1668	.1337	.1116	.0938
	.20	.9654	.9031	.7489	.5957	.4413	.3137	.2240	.1679	.1339	.1110	.0932
	.40	.8967	.8335	.5962	.4947	.3876	.2930	.2193	.1686	.1340	.1100	.0919
R	.60	.4951	.4333	.4411	.3840	.3201	.2583	.2060	.1642	.1324	.1087	.0907
M	.80	.3904	.3432	.3228	.2924	.2564	.2131	.1839	.1528	.1269	.1058	.0890
S	1.00	.2557	.2520	.2412	.2246	.2041	.1815	.1585	.1370	.1174	.1003	.0858
	1.20	.1336	.1315	.1334	.1759	.1636	.1436	.1349	.1199	.1057	.0926	.0809
J	1.40	.1513	.1500	.1464	.1405	.1329	.1239	.1141	.1039	.0937	.0839	.0748
I	1.60	.1213	.1205	.1182	.1144	.1094	.1035	.0963	.0897	.0824	.0751	.0681
T	1.80	.0993	.0988	.0972	.0947	.0914	.0873	.0826	.0776	.0723	.0669	.0616
T	2.00	.0828	.0824	.0813	.0795	.0772	.0744	.0710	.0674	.0635	.0595	.0554
E	2.20	.0700	.0697	.0690	.0677	.0661	.0640	.0615	.0588	.0559	.0529	.0497
R	2.40	.0599	.0598	.0592	.0583	.0571	.0555	.0537	.0517	.0495	.0471	.0447
	2.60	.0519	.0516	.0514	.0507	.0497	.0486	.0472	.0457	.0440	.0422	.0402
	2.80	.0454	.0453	.0450	.0444	.0437	.0428	.0418	.0406	.0393	.0378	.0363
	3.00	.0400	.0399	.0397	.0393	.0387	.0380	.0372	.0363	.0352	.0341	.0329

		2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00
	0.00	.0935	.0755	.0660	.0563	.0488	.0426	.0374	.0333	.0300	.0273	.0249
	.20	.0932	.0752	.0660	.0564	.0487	.0425	.0375	.0333	.0300	.0273	.0248
	.40	.0919	.0775	.0658	.0563	.0487	.0425	.0375	.0334	.0300	.0272	.0248
R	.60	.0907	.0766	.0653	.0561	.0487	.0426	.0376	.0335	.0301	.0272	.0247
M	.80	.0890	.0754	.0646	.0557	.0485	.0425	.0376	.0335	.0301	.0271	.0246
S	1.00	.0858	.0730	.0635	.0551	.0481	.0423	.0375	.0335	.0300	.0271	.0246
	1.20	.0809	.0705	.0616	.0540	.0475	.0420	.0373	.0333	.0299	.0270	.0245
J	1.40	.0748	.0664	.0588	.0522	.0463	.0413	.0369	.0331	.0298	.0269	.0245
I	1.60	.0681	.0615	.0553	.0497	.0446	.0401	.0361	.0326	.0294	.0267	.0243
T	1.80	.0616	.0564	.0514	.0468	.0425	.0386	.0350	.0318	.0289	.0263	.0241
T	2.00	.0554	.0513	.0474	.0436	.0400	.0367	.0336	.0308	.0282	.0258	.0237
E	2.20	.0497	.0466	.0434	.0404	.0374	.0346	.0320	.0295	.0272	.0251	.0231
R	2.40	.0447	.0422	.0397	.0372	.0348	.0325	.0302	.0281	.0261	.0242	.0225
	2.60	.0402	.0383	.0363	.0343	.0323	.0303	.0284	.0266	.0249	.0232	.0217
	2.80	.0363	.0347	.0331	.0315	.0299	.0282	.0266	.0251	.0236	.0222	.0208
	3.00	.0329	.0316	.0303	.0290	.0276	.0262	.0249	.0236	.0223	.0211	.0199

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO E-M PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .2000 T = 1-ALPHA**2 = .9600

AVERAGE IRRADIANCE OVER A CIRCULAR HOLE

HOLE RADIUS

RMS JITTER	0.00	.20	.40	.60	.80	1.00	1.20	1.40	1.60	1.80	2.00
0.00	1.0000	.8477	.8331	.6240	.4442	.3030	.2113	.1601	.1325	.1140	.0977
.20	.9011	.8570	.7346	.5833	.4243	.2980	.2124	.1617	.1326	.1131	.0968
.40	.6353	.6576	.5826	.4733	.3728	.2832	.2107	.1636	.1328	.1115	.0949
.60	.4310	.4655	.4272	.3703	.3065	.2493	.1994	.1605	.1315	.1097	.0929
.80	.3335	.3313	.3115	.2825	.2481	.2126	.1794	.1504	.1263	.1066	.0907
1.00	.2477	.2441	.2339	.2182	.1988	.1775	.1560	.1357	.1172	.1011	.0872
1.20	.1590	.1570	.1813	.1722	.1607	.1475	.1335	.1195	.1060	.0935	.0822
1.40	.1490	.1479	.1444	.1389	.1316	.1231	.1139	.1040	.0943	.0849	.0760
1.60	.1206	.1195	.1176	.1140	.1093	.1036	.0971	.0903	.0832	.0762	.0694
1.80	.0995	.0990	.0975	.0951	.0916	.0879	.0834	.0785	.0733	.0680	.0628
2.00	.0835	.0831	.0821	.0804	.0781	.0753	.0720	.0684	.0646	.0606	.0566
2.20	.0710	.0707	.0700	.0686	.0671	.0650	.0627	.0600	.0571	.0541	.0509
2.40	.0611	.0609	.0603	.0594	.0582	.0567	.0549	.0529	.0506	.0483	.0459
2.60	.0531	.0529	.0525	.0516	.0509	.0497	.0484	.0468	.0451	.0433	.0414
2.80	.0465	.0464	.0461	.0456	.0449	.0440	.0429	.0417	.0404	.0389	.0374
3.00	.0411	.0410	.0405	.0404	.0398	.0391	.0383	.0373	.0363	.0351	.0339

RMS JITTER	2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00
0.00	.0977	.0825	.0695	.0593	.0513	.0447	.0393	.0349	.0314	.0285	.0258
.20	.0968	.0821	.0693	.0593	.0513	.0447	.0394	.0350	.0314	.0284	.0258
.40	.0949	.0809	.0691	.0592	.0512	.0447	.0394	.0350	.0314	.0284	.0258
.60	.0929	.0794	.0682	.0588	.0511	.0447	.0394	.0351	.0314	.0284	.0257
.80	.0907	.0777	.0670	.0582	.0508	.0446	.0394	.0351	.0314	.0283	.0257
1.00	.0872	.0755	.0656	.0572	.0502	.0443	.0392	.0350	.0314	.0283	.0257
1.20	.0822	.0722	.0634	.0555	.0493	.0437	.0383	.0348	.0313	.0283	.0256
1.40	.0760	.0675	.0604	.0533	.0480	.0423	.0383	.0344	.0311	.0281	.0255
1.60	.0694	.0625	.0566	.0512	.0461	.0415	.0375	.0339	.0307	.0279	.0254
1.80	.0628	.0577	.0525	.0481	.0438	.0399	.0363	.0330	.0301	.0274	.0251
2.00	.0566	.0525	.0487	.0449	.0413	.0379	.0348	.0319	.0293	.0268	.0247
2.20	.0509	.0475	.0445	.0416	.0386	.0358	.0331	.0306	.0282	.0261	.0241
2.40	.0459	.0434	.0409	.0384	.0359	.0336	.0313	.0291	.0271	.0251	.0234
2.60	.0414	.0394	.0374	.0353	.0333	.0313	.0294	.0276	.0259	.0241	.0225
2.80	.0374	.0359	.0342	.0325	.0309	.0292	.0276	.0260	.0245	.0230	.0219
3.00	.0339	.0326	.0313	.0299	.0285	.0272	.0258	.0245	.0231	.0219	.0206

A. GIVEN

1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN

THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS "HOLE RADIUS", AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO E-M PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(2*SIGMA**2))/(2*PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .3000 T = 1-ALPHA**2 = .9100

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

HOLE RADIUS

		0.00	.20	.40	.60	.80	1.00	1.20	1.40	1.60	1.80	2.00
	0.00	1.0000	.8473	.8065	.6212	.4407	.2998	.2090	.1591	.1325	.1146	.0985
	.20	.9303	.8056	.7366	.5780	.4210	.2950	.2103	.1608	.1326	.1137	.0976
	.40	.6533	.6553	.5793	.4769	.3700	.2773	.2091	.1629	.1328	.1119	.0956
R	.60	.4753	.4541	.4246	.3684	.3063	.2477	.1983	.1600	.1314	.1101	.0935
M	.80	.3363	.3233	.3093	.2803	.2466	.2115	.1787	.1501	.1263	.1069	.0911
S	1.00	.2463	.2427	.2326	.2171	.1979	.1768	.1555	.1355	.1173	.1013	.0876
	1.20	.1862	.1863	.1806	.1717	.1603	.1472	.1334	.1195	.1061	.0937	.0825
J	1.40	.1407	.1476	.1441	.1397	.1315	.1231	.1138	.1042	.0945	.0851	.0763
I	1.60	.1260	.1198	.1176	.1141	.1093	.1037	.0973	.0905	.0835	.0765	.0697
T	1.80	.0997	.0991	.0977	.0953	.0920	.0881	.0836	.0787	.0736	.0683	.0631
T	2.00	.0837	.0833	.0823	.0806	.0783	.0755	.0723	.0687	.0649	.0609	.0569
E	2.20	.0712	.0710	.0702	.0690	.0674	.0653	.0629	.0603	.0574	.0543	.0512
R	2.40	.0613	.0611	.0606	.0597	.0585	.0569	.0551	.0531	.0509	.0486	.0461
	2.60	.0533	.0532	.0528	.0521	.0512	.0500	.0486	.0471	.0456	.0435	.0416
	2.80	.0468	.0467	.0464	.0458	.0451	.0442	.0432	.0420	.0406	.0392	.0376
	3.00	.0414	.0413	.0410	.0406	.0401	.0394	.0385	.0376	.0365	.0354	.0341

		2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00
	0.00	.0985	.0833	.0703	.0599	.0518	.0452	.0398	.0353	.0317	.0287	.0260
	.20	.0976	.0829	.0702	.0599	.0516	.0452	.0398	.0353	.0317	.0287	.0260
	.40	.0956	.0817	.0693	.0599	.0516	.0452	.0398	.0354	.0317	.0286	.0259
R	.60	.0935	.0800	.0683	.0594	.0516	.0452	.0398	.0354	.0317	.0286	.0259
M	.80	.0911	.0783	.0676	.0587	.0513	.0450	.0398	.0354	.0317	.0286	.0259
S	1.00	.0876	.0759	.0660	.0577	.0506	.0447	.0396	.0353	.0317	.0286	.0259
	1.20	.0825	.0725	.0638	.0562	.0497	.0441	.0393	.0351	.0316	.0285	.0259
J	1.40	.0763	.0692	.0603	.0542	.0483	.0432	.0387	.0347	.0313	.0284	.0258
I	1.60	.0697	.0632	.0571	.0515	.0464	.0419	.0379	.0341	.0309	.0281	.0256
T	1.80	.0631	.0580	.0531	.0484	.0441	.0402	.0366	.0333	.0303	.0277	.0253
T	2.00	.0569	.0529	.0459	.0452	.0416	.0382	.0350	.0321	.0295	.0271	.0249
E	2.20	.0512	.0481	.0443	.0415	.0389	.0360	.0333	.0308	.0285	.0263	.0243
R	2.40	.0461	.0436	.0411	.0386	.0362	.0338	.0315	.0293	.0273	.0253	.0235
	2.60	.0416	.0395	.0375	.0356	.0336	.0316	.0296	.0278	.0260	.0243	.0227
	2.80	.0376	.0360	.0344	.0327	.0311	.0294	.0278	.0262	.0247	.0232	.0218
	3.00	.0341	.0323	.0315	.0301	.0288	.0274	.0260	.0246	.0233	.0220	.0208

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO E-M PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .3162 T = 1-ALPHA**2 = .9000

*** AVERAGE IRRADIANCE OVER A CIRCULAR HOLE ***

HOLE RADIUS

RMS JITTER	0.00	.20	.40	.60	.80	1.00	1.20	1.40	1.60	1.80	2.00
0.00	1.0000	.9445	.7968	.6046	.4208	.2818	.1971	.1545	.1338	.1191	.1037
.20	.8952	.8437	.7245	.5612	.4023	.2786	.1995	.1565	.1336	.1177	.1025
.40	.6698	.6414	.5645	.4605	.3543	.2651	.2039	.1595	.1334	.1150	.0997
.60	.4623	.4466	.4098	.3547	.2946	.2398	.1923	.1577	.1319	.1124	.0968
.80	.3241	.3170	.2987	.2711	.2388	.2059	.1754	.1489	.1269	.1088	.0939
1.00	.2387	.2354	.2259	.2114	.1934	.1738	.1540	.1352	.1182	.1031	.0900
1.20	.1844	.1826	.1773	.1690	.1583	.1461	.1331	.1203	.1073	.0955	.0847
1.40	.1474	.1463	.1431	.1380	.1312	.1232	.1144	.1052	.0960	.0869	.0784
1.60	.1208	.1201	.1180	.1146	.1101	.1045	.0985	.0919	.0851	.0783	.0716
1.80	.1007	.1002	.0986	.0965	.0933	.0895	.0852	.0804	.0753	.0702	.0650
2.00	.0852	.0849	.0839	.0822	.0800	.0772	.0740	.0705	.0667	.0628	.0588
2.20	.0730	.0727	.0723	.0708	.0691	.0671	.0647	.0621	.0592	.0562	.0530
2.40	.0631	.0629	.0624	.0615	.0603	.0587	.0569	.0549	.0527	.0503	.0479
2.60	.0551	.0550	.0545	.0539	.0529	.0518	.0504	.0488	.0471	.0452	.0433
2.80	.0485	.0484	.0481	.0475	.0468	.0459	.0448	.0436	.0422	.0408	.0392
3.00	.0430	.0429	.0427	.0422	.0417	.0410	.0401	.0391	.0380	.0369	.0356

RMS JITTER	2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00
0.00	.1037	.0883	.0741	.0633	.0550	.0483	.0425	.0377	.0337	.0303	.0273
.20	.1025	.0875	.0741	.0634	.0550	.0482	.0425	.0377	.0337	.0303	.0273
.40	.0997	.0859	.0737	.0634	.0550	.0481	.0424	.0377	.0337	.0303	.0273
.60	.0968	.0837	.0725	.0623	.0548	.0480	.0424	.0376	.0336	.0303	.0274
.80	.0939	.0814	.0709	.0619	.0542	.0477	.0422	.0376	.0336	.0303	.0274
1.00	.0900	.0786	.0683	.0605	.0534	.0472	.0419	.0374	.0336	.0303	.0274
1.20	.0847	.0759	.0664	.0586	.0522	.0464	.0414	.0371	.0334	.0302	.0274
1.40	.0784	.0734	.0631	.0565	.0506	.0453	.0407	.0367	.0331	.0300	.0273
1.60	.0716	.0692	.0592	.0536	.0485	.0439	.0397	.0359	.0326	.0297	.0271
1.80	.0650	.0599	.0550	.0504	.0461	.0420	.0383	.0350	.0319	.0292	.0267
2.00	.0588	.0547	.0506	.0470	.0434	.0399	.0367	.0338	.0310	.0285	.0262
2.20	.0530	.0499	.0467	.0436	.0406	.0377	.0343	.0323	.0299	.0277	.0256
2.40	.0479	.0453	.0428	.0403	.0378	.0353	.0330	.0308	.0287	.0267	.0248
2.60	.0433	.0413	.0392	.0371	.0351	.0330	.0311	.0291	.0273	.0256	.0239
2.80	.0392	.0376	.0359	.0342	.0325	.0308	.0291	.0275	.0259	.0244	.0229
3.00	.0356	.0343	.0323	.0315	.0301	.0287	.0273	.0259	.0245	.0232	.0219

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO E-M PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2)/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .4000   T = 1-ALPHA**2 = .8400

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

HOLE RADIUS

RMS JITTER	0.00	.20	.40	.60	.80	1.00	1.20	1.40	1.60	1.80	2.00
0.00	1.0000	.9426	.7905	.5938	.4082	.2710	.1905	.1527	.1354	.1222	.1068
.20	.9316	.8440	.7166	.5503	.3906	.2689	.1937	.1549	.1351	.1205	.1054
.40	.6612	.6324	.5547	.4502	.3448	.2577	.1965	.1561	.1343	.1172	.1022
.60	.4533	.4333	.4007	.3464	.2977	.2339	.1899	.1568	.1325	.1140	.0989
.80	.3169	.3134	.2923	.2655	.2344	.2028	.1737	.1485	.1274	.1101	.0956
1.00	.2345	.2313	.2222	.2082	.1910	.1722	.1533	.1353	.1189	.1043	.0915
1.20	.1824	.1805	.1756	.1676	.1573	.1456	.1331	.1204	.1082	.0967	.0861
1.40	.1469	.1458	.1427	.1377	.1312	.1235	.1150	.1060	.0970	.0882	.0798
1.60	.1211	.1204	.1184	.1151	.1107	.1054	.0994	.0930	.0863	.0796	.0730
1.80	.1016	.1011	.0997	.0974	.0943	.0906	.0863	.0816	.0766	.0715	.0664
2.00	.0863	.0858	.0850	.0834	.0811	.0784	.0753	.0718	.0680	.0641	.0601
2.20	.0742	.0740	.0732	.0720	.0704	.0684	.0660	.0634	.0605	.0575	.0543
2.40	.0644	.0642	.0637	.0628	.0616	.0600	.0582	.0562	.0540	.0516	.0491
2.60	.0564	.0562	.0558	.0551	.0542	.0530	.0516	.0501	.0483	.0465	.0445
2.80	.0497	.0496	.0493	.0488	.0480	.0471	.0460	.0448	.0434	.0419	.0404
3.00	.0442	.0441	.0438	.0434	.0428	.0421	.0413	.0403	.0392	.0380	.0367

RMS JITTER	2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00
0.00	.1068	.0904	.0761	.0652	.0570	.0504	.0446	.0395	.0353	.0317	.0286
.20	.1054	.0899	.0761	.0653	.0570	.0503	.0445	.0395	.0353	.0317	.0286
.40	.1022	.0882	.0756	.0654	.0570	.0501	.0444	.0395	.0353	.0317	.0286
.60	.0989	.0859	.0745	.0649	.0568	.0499	.0442	.0394	.0352	.0317	.0286
.80	.0956	.0834	.0723	.0639	.0562	.0496	.0440	.0392	.0351	.0317	.0287
1.00	.0915	.0804	.0705	.0624	.0552	.0490	.0436	.0390	.0350	.0316	.0286
1.20	.0861	.0766	.0681	.0605	.0539	.0481	.0430	.0386	.0348	.0315	.0286
1.40	.0798	.0719	.0647	.0581	.0522	.0469	.0422	.0381	.0344	.0312	.0284
1.60	.0730	.0667	.0607	.0551	.0500	.0453	.0411	.0373	.0339	.0309	.0282
1.80	.0664	.0613	.0563	.0519	.0474	.0434	.0396	.0362	.0331	.0303	.0278
2.00	.0601	.0561	.0522	.0483	.0447	.0412	.0380	.0349	.0321	.0296	.0272
2.20	.0543	.0511	.0480	.0448	.0418	.0389	.0361	.0334	.0310	.0287	.0266
2.40	.0491	.0465	.0440	.0415	.0389	.0365	.0341	.0318	.0297	.0276	.0257
2.60	.0445	.0424	.0404	.0383	.0362	.0341	.0321	.0301	.0283	.0265	.0248
2.80	.0404	.0387	.0370	.0353	.0335	.0318	.0301	.0285	.0268	.0253	.0238
3.00	.0367	.0354	.0340	.0325	.0311	.0296	.0282	.0268	.0254	.0240	.0227

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO [illegible] PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(2*SIGMA**2))/(2*PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .4472 T = 1-ALPHA**2 = .8000

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

HOLE RADIUS

		0.00	.20	.40	.60	.80	1.00	1.20	1.40	1.60	1.80	2.00
	0.00	1.0000	.9433	.7826	.5607	.3933	.2586	.1837	.1516	.1380	.1258	.1098
	.20	.9377	.8842	.7070	.5373	.3768	.2578	.1876	.1537	.1372	.1238	.1082
	.40	.6509	.6216	.5430	.4362	.3339	.2495	.1920	.1571	.1357	.1196	.1047
R	.60	.4420	.4251	.3961	.3370	.2800	.2284	.1869	.1560	.1333	.1157	.1010
M	.80	.3067	.3024	.2850	.2593	.2296	.1995	.1713	.1481	.1281	.1115	.0975
S	1.00	.2297	.2267	.2161	.2043	.1884	.1706	.1525	.1354	.1197	.1056	.0933
	1.20	.1832	.1785	.1737	.1661	.1563	.1451	.1331	.1210	.1092	.0981	.0878
J	1.40	.1463	.1453	.1423	.1375	.1313	.1239	.1155	.1070	.0983	.0897	.0815
I	1.60	.1215	.1209	.1169	.1157	.1115	.1063	.1005	.0943	.0878	.0812	.0748
T	1.80	.1026	.1021	.1003	.0985	.0956	.0919	.0877	.0831	.0782	.0732	.0681
T	2.00	.0877	.0874	.0864	.0843	.0826	.0800	.0768	.0734	.0697	.0658	.0618
E	2.20	.0758	.0755	.0743	.0736	.0720	.0700	.0677	.0650	.0622	.0592	.0560
R	2.40	.0661	.0659	.0653	.0644	.0632	.0617	.0599	.0579	.0556	.0533	.0508
	2.60	.0580	.0579	.0575	.0564	.0559	.0547	.0533	.0517	.0500	.0481	.0461
	2.80	.0514	.0513	.0509	.0504	.0497	.0487	.0476	.0464	.0450	.0435	.0419
	3.00	.0458	.0457	.0454	.0450	.0444	.0437	.0428	.0418	.0407	.0395	.0382

		2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00
	0.00	.1098	.0925	.0773	.0672	.0596	.0532	.0474	.0421	.0376	.0338	.0305
	.20	.1082	.0920	.0780	.0674	.0595	.0531	.0473	.0421	.0376	.0338	.0305
	.40	.1047	.0905	.0779	.0677	.0595	.0529	.0470	.0420	.0376	.0338	.0305
R	.60	.1010	.0882	.0769	.0673	.0592	.0524	.0467	.0417	.0374	.0337	.0305
M	.80	.0975	.0855	.0752	.0663	.0586	.0520	.0464	.0415	.0373	.0336	.0304
S	1.00	.0933	.0824	.0730	.0647	.0575	.0513	.0459	.0411	.0370	.0335	.0304
	1.20	.0878	.0785	.0701	.0627	.0561	.0503	.0452	.0407	.0367	.0333	.0302
J	1.40	.0815	.0735	.0666	.0601	.0542	.0489	.0442	.0400	.0362	.0329	.0300
I	1.60	.0748	.0685	.0626	.0571	.0519	.0472	.0429	.0391	.0356	.0325	.0297
T	1.80	.0681	.0631	.0583	.0536	.0493	.0452	.0414	.0379	.0347	.0319	.0292
T	2.00	.0618	.0578	.0539	.0501	.0464	.0429	.0396	.0365	.0337	.0310	.0286
E	2.20	.0560	.0529	.0497	.0465	.0434	.0403	.0376	.0351	.0324	.0301	.0279
R	2.40	.0508	.0482	.0456	.0431	.0405	.0380	.0356	.0337	.0311	.0290	.0270
	2.60	.0461	.0440	.0413	.0398	.0377	.0356	.0335	.0315	.0296	.0278	.0260
	2.80	.0419	.0402	.0385	.0367	.0350	.0332	.0315	.0297	.0281	.0265	.0249
	3.00	.0382	.0368	.0354	.0339	.0324	.0310	.0295	.0280	.0266	.0252	.0238

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*,A() I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO E-M PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
   U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(. SIGMA**2)/(. PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
   R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .5000   T = 1-ALPHA**2 = .7500

* AVERAGE IRRADIANCE OVER A CIRCULAR HOLE *

HOLE RADIUS

RMS JITTER	.00	.20	.40	.60	.80	1.00	1.20	1.40	1.60	1.80	2.00
0.00	1.0000	.9373	.7749	.5630	.3792	.2475	.1783	.1514	.1407	.1289	.1118
.20	.8835	.8424	.6976	.5247	.3640	.2480	.1827	.1534	.1394	.1264	.1102
.40	.6408	.6111	.5317	.4263	.3239	.2423	.1833	.1565	.1370	.1215	.1066
.60	.4314	.4177	.3803	.3283	.2731	.2236	.1844	.1554	.1340	.1171	.1027
.80	.3012	.2952	.2784	.2533	.2253	.1967	.1704	.1477	.1287	.1127	.0991
1.00	.2255	.2227	.2144	.2017	.1861	.1631	.1513	.1354	.1203	.1068	.0948
1.20	.1733	.1727	.1720	.1643	.1554	.1447	.1332	.1215	.1101	.0994	.0894
1.40	.1456	.1445	.1413	.1374	.1313	.1242	.1163	.1079	.0995	.0911	.0831
1.60	.1219	.1213	.1194	.1163	.1122	.1072	.1015	.0955	.0892	.0828	.0765
1.80	.1036	.1031	.1018	.0996	.0967	.0932	.0891	.0846	.0798	.0748	.0699
2.00	.0891	.0887	.0873	.0862	.0841	.0815	.0784	.0750	.0713	.0675	.0636
2.20	.0773	.0771	.0764	.0752	.0736	.0717	.0693	.0667	.0639	.0609	.0578
2.40	.0677	.0676	.0670	.0661	.0649	.0634	.0616	.0596	.0574	.0550	.0525
2.60	.0598	.0596	.0592	.0585	.0576	.0564	.0550	.0534	.0516	.0498	.0478
2.80	.0531	.0530	.0525	.0521	.0514	.0504	.0493	.0480	.0466	.0451	.0435
3.00	.0474	.0473	.0471	.0466	.0460	.0453	.0444	.0434	.0423	.0410	.0397

RMS JITTER	2.00	2.20	2.40	2.60	2.80	3.00	3.20	3.40	3.60	3.80	4.00
0.00	.1118	.0955	.0789	.0691	.0622	.0563	.0504	.0442	.0400	.0360	.0326
.20	.1102	.0933	.0793	.0693	.0621	.0563	.0502	.0448	.0400	.0360	.0326
.40	.1066	.0922	.0797	.0697	.0619	.0555	.0498	.0446	.0400	.0360	.0326
.60	.1027	.0900	.0783	.0695	.0616	.0550	.0493	.0443	.0398	.0359	.0325
.80	.0991	.0874	.0773	.0685	.0609	.0544	.0488	.0439	.0396	.0358	.0324
1.00	.0948	.0843	.0750	.0663	.0598	.0536	.0482	.0434	.0392	.0355	.0323
1.20	.0894	.0803	.0721	.0643	.0583	.0525	.0473	.0428	.0388	.0352	.0321
1.40	.0831	.0755	.0686	.0621	.0563	.0510	.0463	.0420	.0382	.0348	.0318
1.60	.0765	.0703	.0645	.0590	.0539	.0492	.0449	.0410	.0374	.0343	.0314
1.80	.0699	.0649	.0601	.0555	.0512	.0471	.0432	.0397	.0365	.0335	.0309
2.00	.0636	.0595	.0557	.0519	.0482	.0447	.0414	.0382	.0353	.0327	.0302
2.20	.0578	.0546	.0514	.0483	.0452	.0422	.0393	.0366	.0340	.0316	.0294
2.40	.0525	.0498	.0473	.0447	.0422	.0396	.0372	.0348	.0326	.0304	.0284
2.60	.0478	.0457	.0435	.0414	.0392	.0371	.0350	.0330	.0310	.0291	.0273
2.80	.0435	.0415	.0401	.0383	.0365	.0347	.0329	.0312	.0295	.0278	.0262
3.00	.0397	.0383	.0369	.0354	.0339	.0324	.0309	.0294	.0279	.0264	.0251

A. GIVEN
1. I(X-XO,Y-YO) = IRRADIANCE DISTRIBUTION CENTERED AT (XO,YO).
2. P(XO,YO)*DXO*DYO = PROBABILITY THAT THE IRRADIANCE IS CENTERED WITHIN (DXO,DYO) OF (XO,YO).

B. THEN
THE CONVOLUTION OF I WITH P DESCRIBES THE EFFECTIVE IRRADIANCE, IEFF, AT A POINT (X,Y). THE INTEGRAL OF IEFF(X,Y) OVER ANY AREA A DIVIDED BY THE AREA OF A IS THE AVERAGE IRRADIANCE OVER A, IAV.

C. IN THE ABOVE TABLE OF IAV, THE AREA A IS A CIRCLE WHICH IS CENTERED AT (0,0) AND WHOSE RADIUS IS *HOLE RADIUS*, AND I(X,Y) = I(R) = NORMALIZED FRAUNHOFFER PATTERN DUE TO I-1 PLANE WAVE PASSING THROUGH AN ANNULAR APERTURE, WHERE

1. I(X,Y) = U**2
U = (2*J1(PI*R)-2*ALPHA*J1(PI*ALPHA*R))/PI/R/(1-ALPHA**2)
2. P(X,Y) = EXP(-(X**2+Y**2)/(SIGMA**2))/(PI*SIGMA**2)
3. D = OUTER DIAMETER OF ANNULAR APERTURE
4. ALPHA*D = INNER DIAMETER OF ANNULAR APERTURE
5. WL = WAVELENGTH OF PLANE WAVE
6. F = DISTANCE FROM APERTURE TO FRAUNHOFFER PLANE
7. S = DISTANCE OF ANY POINT IN THE FRAUNHOFFER PLANE FROM THE POINT (0,0) OF THE FRAUNHOFFER PLANE
8. R**2 = X**2+Y**2. R, X, AND Y ARE DIMENSIONLESS.
R = S*D/(F*WL)
9. J1 = FIRST ORDER BESSEL FUNCTION OF THE FIRST KIND

ALPHA = .5477 T = 1-ALPHA**2 = .7000

APPENDIX D

FORTRAN PROGRAM JITTRAN

```
      [illegible]
      [illegible]
C
COMMENT          [illegible]
C
      EXTERNAL ALOG10
      REAL NTEGRAL
      DIMENSION G(4097),TITLE(31),V(11),N(11),W(11)
      COMMON SIGMA,ALPHA
      DATA TITLE/31*1H /
      TITLE( 3)=1HR
      TITLE( 6)=1HM
      TITLE( 9)=1HS
      TITLE(14)=1HJ
      TITLE(17)=1HI
      TITLE(20)=1HT
      TITLE(23)=1HT
      TITLE(26)=1HE
      TITLE(29)=1HR
  1[illegible] FORMAT(1H1/* HOLE RADIUS AND SIGMA ARE IN UNITS OF [illegible]
     1[illegible] F=FOCAL LENGTH, LAMBDA=WAVELENGTH, [illegible] DIAMETER OF
     2APERTURE.*/* ALPHA=RATIO OF INNER DIAMETER OF APERTURE TO OUTER
     3DIAMETER.    ALPHA=*F[illegible]    [illegible]
     [illegible]
  110 FORMAT([illegible])
  120 FORMAT([illegible])
  130 FORMAT([illegible])
  140 FORMAT([illegible])
  1[illegible] FORMAT(* JTOT IS [illegible])
C
COMMENT      [illegible] INTEGRAL OVER CIRCLE.
COMMENT      RADIUS [illegible] CIRCLE = ROMAX. [illegible]
COMMENT      IEFF=CONVOLUTION OF IRRADIANCE [illegible]
COMMENT      IEFF=[illegible]. [illegible] = SUBROUTINE FOR INTEGRATION.
COMMENT      [illegible] = INTEGRAL OF IEFF OVER A CIRCLE
COMMENT      IAV = AVG IRRADIANCE OVER CIRCLE = [illegible] OF CIRCLE
COMMENT      [illegible] THE INTEGRAND BY [illegible]. THEN THE VALUE OF THE
COMMENT    INTEGRAND IN CIRCULAR COORDINATES IS [illegible]
COMMENT    [illegible]. THE AVERAGE VALUE OF [illegible] ACROSS A CIRCLE
COMMENT    [illegible] RADIUS IS [illegible] = THE INTEGRAL [illegible]
COMMENT    FACTORS OF PI CANCEL AND CONSEQUENTLY [illegible] APPEAR EXPLICITLY
COMMENT    IN THE ALGORITHM.
COMMENT        ALPHA=RATIO OF INNER DIAMETER TO OUTER DIAMETER OF ANNULUS.
COMMENT        RO=DISTANCE BETWEEN CENTER OF [illegible] AND CENTER OF [illegible].
COMMENT        DRO=INCREMENT IN RO FOR INTEGRATION.
COMMENT        ROMAX=RADIUS OF [illegible]
COMMENT        ROMAX1 = MIN VALUE OF ROMAX ON A GIVEN [illegible] OF OUTPUT.
COMMENT        SIGMA=STANDARD DEVIATION OF [illegible] DISTRIBUTION.
COMMENT        DSIG=STEP SIZE IN SIGMA.
COMMENT        V(I)=11-PLACE ARRAY USED TO OUTPUT ROWS OF 11 CHARACTERS.
COMMENT        JTOT = TOTAL NO. POINTS AT WHICH THE INTEGRAND [illegible] IS
COMMENT    EVALUATED IN ORDER TO CALCULATE THE INTEGRAL.
COMMENT        JL=LOWER VALUE OF JTOT [illegible] SEVERAL DIFFERENT [illegible]
COMMENT      K = COUNTER. TELLS WHICH ROW OF OUTPUT YOU ARE ON. USED IN
COMMENT    CONJUNCTION WITH FORMAT 1[illegible] AND TITLE(K)
COMMENT        ONE ROW OF OUTPUT CONSISTS OF 11 VALUES OF IAV. IAV(ROMAX1),
COMMENT    IAV(ROMAX1+DROMAX),IAV(ROMAX1+[illegible]ROMAX),....,IAV(ROMAX1+10*DROMAX).
COMMENT        ON A GIVEN ROW SIGMA IS A CONSTANT
COMMENT        FROM ROW TO ROW SIGMA IS INCREMENTED BY DSIG
COMMENT      ROWS OF DATA ARE OUTPUT AS LONG AS SIGMA.LT.SIGSTOP
COMMENT
```

```
C     USER MUST INITIALIZE THE FOLLOWING DATA...ALPHA,T,SIGMA,DSIG,SIGSTOP,ROMAX1,
C     DROMAX, AND M.
C
        ALPHA =
        T = 1.-ALPHA*ALPHA
        SIGMA= .
        SIGSTOP=1.5
        DSIG=.05
        ROMAX1=0.
        DROMAX = .05
        M=
C     END OF INITIALIZATION
C     IDEL = NO. OF PTS. ON EACH DROMAX INTERVAL FOR WHICH INTEGRAND,G(RO),
C     IS CALCULATED.
        IDEL = 2
C     11 COLUMNS OF DATA
        DO 1 I=1,11
      1 V(I)=ROMAX1+(I-1)*DROMAX
        PRINT 100,ALPHA,T
        PRINT 110
        PRINT 110,(V(I),I=1,11)
        PRINT120
        PUNCH 140,ALPHA
      2 I=11
        M=M+1
C     11 COLUMNS OF DATA
        ROMAX = ROMAX1+DROMAX*10.
        DRO=DROMAX/IDEL
        JTOT=INT(.1+ROMAX/DRO)
        JL=0
C     FILL IN EVERY OTHER VALUE OF G(RO)
        DO 3 J=JL,JTOT,2
        RO=J*DRO
      3 G(2*J+1)=INTEGRAL(RO)*RO
      4 SUM1=SUM2=0.
COMMENT           INITIALIZE SUM1
        JJ=2*JTOT
        DO 5 J=JL,JJ,4
      5 SUM1=SUM1+G(J+1)
C     SUM1 INITIALIZED.   SUM1=SUM OF EVERY OTHER VALUE OF G(RO)
        JL=2
        JDEL=4
      6 IDEL=2*IDEL
        DRO = DRO/2.
        JTOT=2*JTOT
      7 SUM1=SUM1+SUM2
        SUM2=0.
COMMENT           CALCULATE SUM2
        DO 8 J=JL,JTOT,JDEL
        RO=J*DRO
        G(J+1)= INTEGRAL(RO)*RO
      8 SUM2=SUM2+G(J+1)
C
COMMENT           SIMPSON'S RULE
C
        V(I)=2.*SUM1+4.*SUM2-G(1)-G(1+JTOT)
        V(I)=V(I)+DRO*V(I)*2./6.
COMMENT           IF JL=4. THEN THIS IS THE FIRST EVALUATION OF V(I) FOR A GIVEN
COMMENT     VALUE OF ROMAX
COMMENT           IF THIS IS THE FIRST EVALUATION OF V(I), THEN LET VOLD = V(I) .
COMMENT     AND REEVALUATE V(I) USING EVERY VALUE OF [illegible] EVERY OTHER [illegible]
```

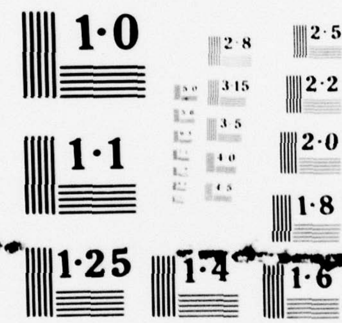
```
      IF(2-JL)9,9,10
    9 CHK=V(I)
      JL=1
      JDEL=2
      GOTO 7
   10 ERR= ABS(V(I)-CHK)
      IF(.001*V(I)-ERR)11,14,14
C     CHECK TO ASSURE INTEGRAL OVER RO DOES NOT NEED A LOT OF POINTS.
   11 IF(2*JTOT-4097)12,12,24
   24 PRINT 180,RO
      GOTO 14
COMMENT          SHIFT EACH VALUE IN G(J+1) TO G(2*J+1).
   12 DO 13 J=1,JTOT
      J1=JTOT-J+2
      J2=2*J1-1
   13 G(J2)=G(J1)
      CHK=V(I)
      GO TO 6
   14 V(I)=2*V(I)/ROMAX/ROMAX
   15 CONTINUE
   16 CONTINUE
      I=I-1
      IF(I-1)19,17,17
COMMENT          DO THIS IF ROMAX>0 AND I≠0
   17 JL=0
      JTOT=(JTOT-IDEL)/2
      ROMAX=ROMAX-DROMAX
      IDEL=IDEL/2
      DRO=DRO*2.
      IF(ROMAX-.001)18,4,4
COMMENT          DO THIS IF ROMAX=0 AND I=1.   (ROMAX=0 IMPLIES I=1.)
   18 V(I)=NTEGRAL(0.)
      GO TO 15
COMMENT          OUTPUT ONE LINE OF V(I).
   19 PRINT 130,TITLE(M),SIGMA,(V(I),I=1,11)
      PUNCH 140,SIGMA,(V(I)        ,I=1,11)
      IDEL=IDEL/2
   22 SIGMA=SIGMA + DSIG
      IF(SIGMA-SIGSTOP)2,23,23
   23 PRINT 120
   25 CONTINUE
      END
```

AD-A065 978 AIR FORCE WEAPONS LAB KIRTLAND AFB N MEX F/G 20/5
GAUSSIAN JITTER OF A FOCUSED BEAM OF LIGHT.(U)
APR 76 W T WHITE, J P BAUMGARDNER, D A HOLMES

UNCLASSIFIED AFWL-TR-76-66 SBIE-AD-E200 250 NL

2 OF 2
ADA
065978

END
DATE
FILMED
5-79
DDC



NATIONAL BUREAU OF STANDARDS
MICROCOPY RESOLUTION TEST CHART

```
      REAL FUNCTION  NTEGRAL(X)
C    THIS ALGORITHM ADAPTED FROM MATHOUPOGA. A NESTED GAUSSIAN QUADRATURE
C    MULTIPLE INTEGRAL SCHEME.  AIR FORCE WEAPONS LABORATORY.KIRTLAND AFB.NM
C
COMMENT        NTEGRAL(X) INTEGRATES FROM R=0 AND THETA=0 TO R=4*SIGMA AND
C          THETA= PI .  THE INTEGRAND IS A FUNCTION F(R,THETA,X) TIMES A
C          PROBABILITY DISTRIBUTION P(R).  SINCE THE INTEGRAL IS  PERFORMED
C          IN ROTATIONAL COORDINATES. THERE IS AN ADDITIONAL FACTOR OF R IN
COMMENT THE INTEGRAND.
      COMMON SIGMA
      DIMENSION AR(130).ANSR(130).R(100).RX(100).GR(100)
      DIMENSION AT(130),ANST(130).T(100).RT(100).GT(100)
      DATA GR/-.9602898564975363,-.7966664774136267,-.5255324099163290,
     1-.1834346424956498,.1834346424956498,.5255324099163290,
     2.7966664774136267,.9602898564975363/
      DATA RX/.1012285362903763,.2223810344533745,.3137066458778873,
     1.3626837833783620,.3626837833783620,.3137066458778873,
     2.2223810344533745   ,.1012285362903763/
      DATA GT/-.9602898564975363,-.7966664774136267,-.5255324099163290,
     1-.1834346424956498,.1834346424956498,.5255324099163290,
     2.7966664774136267,.9602898564975363/
      DATA RT/.1012285362903763,.2223810344533745,.3137066458778873,
     1.3626837833783620,.3626837833783620,.3137066458778873,
     2.2223810344533745   ,.1012285362903763/
      NTEGRAL=0.
      IF(SIGMA-.01)10.10.20
C
C          FOR SIGMA NEAR ZERO. THE PROBABILITY BECOMES APPROXIMATELY A DIRAC
C    DELTA FUNCTION.  THE INTEGRAL OF F(X)*P(X) = F(0).
C
   10 NTEGRAL=F(0.,0.,X)
       RETURN
   20 R1=-SIGMA
C
C          BREAK INTEGRAL INTO 4 INTEGRALS FROM 0 TO 4*SIGMA.
C
      DO 29 I=1.4
COMMENT
COMMENT        BEGIN INTEGRAL-R
COMMENT
      R1=R1+SIGMA
      R2=R1+SIGMA
      CHKR=0.
      NR=1
      AR(1)=R1
   21 RN=NR
      SUMR=0.
      HR=(R2-R1)/RN
      DO22 IR=1.NR
      RI=IR
   22 AR(IR+1)=R1+RI*HR
      DO 25 IRI=1.NR
      ANSR(IRI)=0.
      DO 23 JR=1.8
      R(JR)=(AR(IRI+1)-AR(IRI))*GR(JR)+AR(IRI+1)+AR(IRI)
   23 R(JR)=R(JR)/2.
```

```
      DO 24 IR=1,N
      RR=R(IR)
COMMENT
COMMENT           BEGIN INTEGRAL THETA
COMMENT
      T1=0.
      T2=3.14159265358
      CHKT=0.
      NT=1
      AT(1)=T1
   31 TN=NT
      SUMT=0.
      HT=(T2-T1)/TN
      DO 32 II=1,NT
      TI=II
   32 AT(II+1)=T1+TI*HT
      DO 35 ITI=1,NT
      ANST(ITI)=0.
      DO 33 JT=1,8
      T(JT)=(AT(ITI+1)-AT(ITI))*U(JT)+AT(ITI+1)+AT(ITI)
   33 T(JT)=T(JT)/2.
      DO 34 IT=1,8
      TT=T(IT)
   34 ANST(ITI)=ANST(ITI)+W(IT)*F(RR,TT,K)
   35 ANST(ITI)=(AT(ITI+1)-AT(ITI))/2.*ANST(ITI)
      DO 36 IT=1,NT
   36 SUMT=SUMT+ANST(IT)
      ERT=ABS(SUMT-CHKT)
      IF(ERT-.0001*SUMT)39,39,37
   37 IF(ERT.LT.1E-17)GO TO 39
      NT=2*NT
      CHKT=SUMT
      IF(NT-130)31,31,38
   38 RIN=RMIN IN T-I
      PRINT 111,ERT,RIN,SIGMA,U,R1,R2
  111 FORMAT(21H ERROR IS OF ORDER  ,E10.3,2X,*RHO=1*,*SIGMA=*,F5.2,
     110X,*U=*,F5.2,15X,*R1=*,F5.2,10X,*R2=*,F5.2)
   39 CONTINUE
COMMENT
COMMENT END INTEGRAL THETA  .  ..
COMMENT
   24 ANSR(IR)=ANSR(IR)+W(IR)*SUMT*RR
   25 ANSR(IR)=(AR(IR+1)-AR(IR))/2.*ANSR(IR)
      DO 26 IR=1,NR
   26 SUMR=SUMR+ANSR(IR)
      ERR=ABS(SUMR-CHKR)
      IF(ERR-.0001*SUMR)29,29,27
   27 IF(ERR.LT.1E-17)GO TO 29
      NR=2*NR
      CHKR=SUMR
      IF(NR-130)21,21,28
   28 RIN=RMIN IN R
      PRINT 111,ERR,RIN,SIGMA,U,R1,R2
   29 NTEGRAL=NTEGRAL+SUMR
C
COMMENT           END NTEGRAL-R
C
C         SINCE THE INTEGRAL INVOLVED VALUES OF THETA ONLY FROM ZERO TO PI,
C       AND SINCE THE INTEGRAL IS SYMMETRIC ABOUT THETA = PI, AND SINCE WE WANT TO
C       INTEGRATE FROM THETA = ZERO TO THETA = 2*PI, WE INTRODUCE A FACTOR OF TWO
C       INTO THE INTEGRAL.
```

```
C
      NIEGRAL=2.*INTEGRAL
      RETURN
      END




      FUNCTION F(R,THETA,RO)
C     F=INSTANTANEOUS IRRADIANCE AT (R,THETA) WHEN CENTER OF BEAM IS AT
C     (RO,THETAO).  THETAO = 0.
      COMMON F(1) ,ALPHA
      EXTERNAL COS
      R1=3.14159265358*SQRT(R*R+[illegible]-2*R*RO*COS(THETA))
      R2=R1*ALPHA
      F=BESSINK(R1)
      IF(ALPHA.NE.0.)F=(F-BESSINK(R2)*ALPHA*ALPHA)/(1.-ALPHA*ALPHA)
      F=F*F
      RETURN
      END


      FUNCTION BESSINK(X)
C     COMPUTES J1(X)/X.  MODIFIED FROM ALGORITHM ON PAGE 370 OF NBS HANDBOOK OF
C     MATHEMATICAL FUNCTIONS, EDITED BY M ABRAMOWITZ ET AL.
      X=ABS(X)
      IF(X-4.2)1,1,2
    1 Z=X*X
      BESSINK=1.-.125*Z*(1.-Z/24.*(1.-Z/[illegible].*(1.-Z/[illegible].*(1.-Z/120.*(
     11.-Z/168.*(1.-Z/224.*(1.-Z/[illegible].*(1.-Z/360.*(1-Z/440.*(1.-Z/528.*(
     21.-Z/624.*(1.-Z/728.*(1.-Z/[illegible].*(1.-Z/960.*(1.-Z/1088.*(1.-Z/1224.
     3*(1.-Z/1368.*(1.-Z/152[illegible].*(1.-Z/1680.))))))))))))))))))    )
      RETURN
    2 Z=X/3.
      F1=.79788456+(.00000156+(.01659667+(.00017105-(.00249511-(
     1.00113653-.00020033/Z)/Z)/Z)/Z)/Z)/Z
      T1=3.*Z-2.35619449+(.12499612+(.0000565-(.00637879-(.00074348+
     1(.00079824-.00029166/Z)/Z)/Z)/Z)/Z)/Z
      Z=2./X/SQRT(X)
      BESSINK=Z*F1*COS(T1)
      RETURN
      END


      FUNCTION P(R)
C     COMPUTES 1-D GAUSSIAN PROBABILITY DENSITY
      COMMON SIGMA
      SIGMA2=SIGMA*SIGMA
      P=EXP(-R*R/SIGMA2)/3.14159265358/SIGMA2
      RETURN
      END
```

APPENDIX E

$\langle I \rangle$ AS A FUNCTION OF ρ AND σ FOR THE CASE $\alpha = 0$

RADIAL PROFILE OF IEFF(RHO) AT EVENLY-SPACED VALUES OF RHO

****

IEFF(RHO + .05M)

RHO		M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0	+	1.00000	.99385	.97558	.94575	.90527	.85535	.79745	.73324	.66451	.59311
.5	+	.52086	.44350	.38064	.31568	.25577	.20181	.15440	.11387	.08027	.05339
1.0	+	.03283	.01799	.00814	.00249	.00018	.00037	.00229	.00518	.00846	.01152
1.5	+	.01429	.01622	.01729	.01746	.01680	.01544	.01355	.01133	.00897	.00667
2.0	+	.00457	.00281	.00145	.00055	.00008	.00002	.00029	.00081	.00148	.00220
2.5	+	.00289	.00348	.00390	.00412	.00414	.00396	.00361	.00313	.00257	.00198
3.0	+	.00141	.00090	.00049	.00020	.00004	.00000	.00008	.00026	.00049	.00076
3.5	+	.00103	.00126	.00145	.00156	.00160	.00156	.00145	.00128	.00107	.00094
4.0	+	.00061									

SIGMA=0.0 T=1.00

****

IEFF(RHO + .05M)

RHO		M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0	+	.90889	.90365	.88809	.86266	.82811	.78542	.73579	.69059	.62129	.55943
.5	+	.49652	.43402	.37328	.31549	.26167	.21260	.16886	.13077	.09846	.07183
1.0	+	.05061	.03438	.02258	.01460	.00979	.00745	.00694	.00767	.00910	.01079
1.5	+	.01239	.01366	.01443	.01464	.01427	.01339	.01211	.01054	.00882	.00708
2.0	+	.00545	.00402	.00285	.00199	.00145	.00120	.00121	.00143	.00178	.00220
2.5	+	.00263	.00300	.00329	.00345	.00347	.00336	.00313	.00291	.00241	.00199
3.0	+	.00157	.00119	.00086	.00062	.00046	.00039	.00041	.00049	.00063	.00079
3.5	+	.00095	.00111	.00123	.00131	.00134	.00132	.00124	.00113	.00099	.00082
4.0	+	.00065									

SIGMA= .2 T=1.00

****

IEFF(RHO + .05M)

RHO		M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0	+	.70622	.70286	.69285	.67645	.65408	.62630	.59380	.55736	.51783	.47612
.5	+	.43312	.38372	.34676	.30500	.26511	.22767	.19310	.16174	.13379	.10930
1.0	+	.08826	.07054	.05594	.04416	.03491	.02784	.02259	.01882	.01620	.01443
1.5	+	.01325	.01245	.01186	.01134	.01081	.01022	.00955	.00880	.00799	.00714
2.0	+	.00631	.00551	.00479	.00416	.00364	.00324	.00295	.00275	.00264	.00259
2.5	+	.00258	.00259	.00259	.00258	.00255	.00247	.00237	.00223	.00207	.00189
3.0	+	.00170	.00152	.00136	.00121	.00109	.00100	.00095	.00091	.00091	.00091
3.5	+	.00093	.00095	.00097	.00098	.00098	.00096	.00093	.00088	.00082	.00076
4.0	+	.00069									

SIGMA= .4 T=1.00

****

IEFF(RHO + .05M)

RHO		M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0	+	.60134	.59883	.59137	.57913	.56239	.54153	.51702	.48939	.45924	.42717
.5	+	.39383	.35984	.32581	.29230	.25981	.22878	.19959	.17250	.14774	.12541
1.0	+	.10557	.08819	.07320	.06045	.04978	.04098	.03383	.02811	.02359	.02005
1.5	+	.01731	.01518	.01351	.01218	.01109	.01015	.00931	.00854	.00782	.00712
2.0	+	.00647	.00585	.00528	.00477	.00431	.00392	.00359	.00331	.00309	.00292
2.5	+	.00277	.00266	.00256	.00247	.00238	.00228	.00218	.00207	.00195	.00183
3.0	+	.00170	.00158	.00147	.00136	.00127	.00118	.00112	.00106	.00102	.00099
3.5	+	.00097	.00095	.00093	.00092	.00090	.00087	.00084	.00081	.00077	.00073
4.0	+	.00069									

SIGMA= .5 T=1.00

****

RADIAL PROFILE OF IEFF(RHO) AT EVENLY-SPACED VALUES OF RHO

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.50722	.50540	.49999	.49110	.47891	.46367	.44566	.42526	.40284	.37882
.5 +	.35361	.32766	.30136	.27512	.24931	.22426	.20024	.17751	.15623	.13656
1.0 +	.11858	.10232	.08780	.07495	.06372	.05400	.04567	.03861	.03268	.02773
1.5 +	.02364	.02027	.01750	.01523	.01336	.01180	.01051	.00941	.00847	.00765
2.0 +	.00694	.00631	.00574	.00524	.00479	.00439	.00404	.00373	.00345	.00322
2.5 +	.00302	.00284	.00267	.00253	.00239	.00227	.00215	.00204	.00193	.00182
3.0 +	.00172	.00162	.00153	.00144	.00136	.00128	.00122	.00116	.00110	.00106
3.5 +	.00101	.00098	.00094	.00091	.00088	.00085	.00081	.00078	.00075	.00072
4.0 +	.00069									

SIGMA= .6 T=1.00

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.36109	.36016	.35739	.35281	.34650	.33855	.32908	.31824	.30619	.29306
.5 +	.27907	.26439	.24921	.23371	.21807	.20247	.18704	.17195	.15730	.14322
1.0 +	.12980	.11709	.10517	.09406	.08379	.07435	.06574	.05794	.05092	.04464
1.5 +	.03905	.03411	.02976	.02595	.02262	.01974	.01724	.01509	.01324	.01165
2.0 +	.01028	.00911	.00810	.00723	.00649	.00584	.00529	.00481	.00439	.00402
2.5 +	.00369	.00341	.00315	.00293	.00273	.00255	.00238	.00223	.00210	.00197
3.0 +	.00186	.00175	.00166	.00157	.00149	.00141	.00134	.00127	.00121	.00115
3.5 +	.00110	.00105	.00100	.00096	.00091	.00088	.00084	.00080	.00077	.00074
4.0 +	.00071									

SIGMA= .8 T=1.00

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.26357	.26308	.26161	.25918	.25582	.25156	.24645	.24056	.23393	.22665
.5 +	.21878	.21041	.20162	.19249	.18311	.17356	.16392	.15426	.14465	.13518
1.0 +	.12588	.11682	.10806	.09962	.09154	.08385	.07657	.06971	.06329	.05729
1.5 +	.05172	.04658	.04196	.03753	.03358	.02999	.02675	.02382	.02120	.01885
2.0 +	.01675	.01488	.01323	.01176	.01047	.00933	.00833	.00744	.00667	.00599
2.5 +	.00540	.00487	.00442	.00401	.00366	.00335	.00307	.00283	.00261	.00242
3.0 +	.00225	.00209	.00195	.00183	.00172	.00161	.00152	.00143	.00136	.00128
3.5 +	.00122	.00115	.00110	.00104	.00100	.00095	.00091	.00087	.00083	.00079
4.0 +	.00076									

SIGMA=1.0 T=1.00

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.19873	.19846	.19763	.19626	.19437	.19196	.18905	.18567	.18185	.17762
.5 +	.17301	.16806	.16280	.15728	.15153	.14560	.13952	.13334	.12709	.12081
1.0 +	.11455	.10832	.10216	.09611	.09019	.08442	.07882	.07341	.06821	.06323
1.5 +	.05848	.05396	.04968	.04564	.04185	.03829	.03497	.03187	.02900	.02635
2.0 +	.02390	.02165	.01959	.01771	.01599	.01442	.01300	.01172	.01056	.00951
2.5 +	.00857	.00773	.00697	.00630	.00569	.00515	.00467	.00424	.00385	.00351
3.0 +	.00321	.00293	.00269	.00248	.00228	.00211	.00196	.00182	.00169	.00158
3.5 +	.00148	.00139	.00130	.00123	.00116	.00109	.00104	.00098	.00093	.00089
4.0 +	.00084									

SIGMA=1.2 T=1.00

RADIAL PROFILE OF IEFF(RHO) AT EVENLY-SPACED VALUES OF RHO

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.15443	.15426	.15377	.15296	.15182	.15037	.14862	.14658	.14425	.14168
.5 +	.13864	.13578	.13251	.12904	.12540	.12161	.11769	.11365	.10953	.10534
1.0 +	.10110	.09684	.09256	.08830	.08406	.07986	.07573	.07167	.06769	.06381
1.5 +	.06004	.05639	.05296	.04946	.04619	.04306	.04008	.03724	.03454	.03198
2.0 +	.02957	.02730	.02517	.02317	.02130	.01955	.01793	.01642	.01503	.01374
2.5 +	.01254	.01145	.01044	.00952	.00867	.00790	.00720	.00655	.00597	.00544
3.0 +	.00496	.00452	.00413	.00377	.00345	.00316	.00289	.00266	.00244	.00225
3.5 +	.00207	.00192	.00177	.00165	.00153	.00143	.00133	.00125	.00117	.00110
4.0 +	.00103									

SIGMA=1.4 T=1.00

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.12313	.12303	.12272	.12221	.12149	.12058	.11947	.11817	.11669	.11504
.5 +	.11322	.11125	.10913	.10686	.10448	.10197	.09937	.09666	.09388	.09103
1.0 +	.08812	.08517	.08218	.07916	.07613	.07311	.07009	.06708	.06411	.06117
1.5 +	.05827	.05543	.05264	.04992	.04726	.04468	.04218	.03976	.03743	.03518
2.0 +	.03302	.03095	.02897	.02708	.02528	.02357	.02194	.02041	.01896	.01759
2.5 +	.01631	.01510	.01397	.01291	.01192	.01099	.01013	.00933	.00859	.00791
3.0 +	.00727	.00668	.00614	.00564	.00518	.00476	.00437	.00402	.00370	.00340
3.5 +	.00313	.00288	.00265	.00244	.00225	.00208	.00193	.00178	.00165	.00153
4.0 +	.00143									

SIGMA=1.6 T=1.00

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.10033	.10026	.10005	.09972	.09924	.09864	.09791	.09705	.09607	.09497
.5 +	.09375	.09243	.09100	.08948	.08786	.08616	.08437	.08251	.08059	.07860
1.0 +	.07656	.07448	.07235	.07020	.06802	.06582	.06360	.06139	.05917	.05696
1.5 +	.05476	.05257	.05041	.04828	.04618	.04412	.04209	.04011	.03817	.03629
2.0 +	.03445	.03267	.03095	.02928	.02766	.02611	.02462	.02318	.02181	.02050
2.5 +	.01924	.01804	.01690	.01582	.01479	.01381	.01289	.01202	.01120	.01043
3.0 +	.00970	.00901	.00837	.00777	.00721	.00668	.00620	.00574	.00531	.00492
3.5 +	.00455	.00421	.00390	.00360	.00334	.00309	.00285	.00264	.00245	.00227
4.0 +	.00210									

SIGMA=1.8 T=1.00

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.08324	.08319	.08306	.08283	.08250	.08209	.08159	.08100	.08032	.07957
.5 +	.07873	.07781	.07682	.07576	.07463	.07343	.07218	.07086	.06950	.06808
1.0 +	.06663	.06513	.06359	.06202	.06043	.05881	.05717	.05552	.05386	.05219
1.5 +	.05052	.04885	.04718	.04553	.04388	.04225	.04064	.03904	.03747	.03593
2.0 +	.03442	.03293	.03148	.03006	.02867	.02732	.02601	.02474	.02351	.02232
2.5 +	.02116	.02005	.01898	.01795	.01696	.01601	.01510	.01423	.01340	.01261
3.0 +	.01185	.01113	.01045	.00980	.00918	.00860	.00805	.00753	.00704	.00658
3.5 +	.00614	.00573	.00535	.00498	.00465	.00433	.00403	.00375	.00349	.00325
4.0 +	.00303									

SIGMA=2.0 T=1.00

RADIAL PROFILE OF IEFF(RHO) AT EVENLY-SPACED VALUES OF RHO

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.07014	.07010	.07001	.06984	.06962	.06932	.06897	.06855	.06807	.06753
.5 +	.06634	.06529	.06558	.06482	.06401	.06315	.06224	.06129	.06030	.05927
1.0 +	.05821	.05711	.05598	.05482	.05364	.05243	.05120	.04996	.04870	.04743
1.5 +	.04615	.04487	.04358	.04229	.04100	.03972	.03844	.03717	.03591	.03466
2.0 +	.03343	.03221	.03100	.02982	.02866	.02752	.02640	.02530	.02424	.02319
2.5 +	.02217	.02118	.02022	.01928	.01838	.01750	.01665	.01583	.01504	.01427
3.0 +	.01354	.01283	.01215	.01150	.01088	.01028	.00971	.00916	.00864	.00815
3.5 +	.00767	.00722	.00680	.00639	.00601	.00564	.00530	.00497	.00466	.00437
4.0 +	.00410									

SIGMA=2.2 T=1.00

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.05988	.05985	.05978	.05967	.05950	.05929	.05903	.05873	.05838	.05799
.5 +	.05755	.05708	.05656	.05600	.05541	.05477	.05411	.05340	.05267	.05191
1.0 +	.05111	.05029	.04944	.04857	.04768	.04677	.04583	.04489	.04392	.04295
1.5 +	.04196	.04097	.03996	.03896	.03794	.03693	.03591	.03490	.03389	.03288
2.0 +	.03188	.03089	.02990	.02893	.02796	.02701	.02607	.02514	.02423	.02334
2.5 +	.02246	.02160	.02075	.01993	.01912	.01834	.01757	.01682	.01610	.01539
3.0 +	.01471	.01405	.01340	.01278	.01218	.01160	.01104	.01050	.00998	.00948
3.5 +	.00900	.00854	.00810	.00767	.00727	.00688	.00651	.00615	.00582	.00549
4.0 +	.00519									

SIGMA=2.4 T=1.00

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.05170	.05168	.05163	.05154	.05142	.05126	.05107	.05085	.05059	.05030
.5 +	.04997	.04962	.04923	.04881	.04837	.04789	.04739	.04686	.04631	.04573
1.0 +	.04513	.04450	.04386	.04319	.04251	.04181	.04109	.04036	.03961	.03885
1.5 +	.03809	.03731	.03652	.03573	.03493	.03412	.03331	.03250	.03169	.03087
2.0 +	.03006	.02925	.02845	.02765	.02685	.02606	.02527	.02450	.02373	.02297
2.5 +	.02222	.02148	.02076	.02004	.01934	.01865	.01797	.01731	.01666	.01603
3.0 +	.01541	.01480	.01421	.01364	.01308	.01254	.01201	.01150	.01100	.01052
3.5 +	.01005	.00960	.00916	.00874	.00834	.00795	.00757	.00720	.00686	.00652
4.0 +	.00620									

SIGMA=2.6 T=1.00

IEFF(RHO + .05M)

RHO	M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0 +	.04509	.04507	.04503	.04496	.04487	.04475	.04461	.04444	.04424	.04402
.5 +	.04377	.04350	.04321	.04289	.04255	.04218	.04180	.04140	.04097	.04052
1.0 +	.04006	.03958	.03908	.03856	.03803	.03749	.03693	.03636	.03577	.03519
1.5 +	.03457	.03396	.03333	.03270	.03206	.03142	.03077	.03012	.02946	.02880
2.0 +	.02814	.02748	.02682	.02616	.02551	.02485	.02420	.02355	.02290	.02227
2.5 +	.02163	.02101	.02038	.01977	.01916	.01857	.01798	.01740	.01683	.01627
3.0 +	.01572	.01518	.01465	.01413	.01362	.01312	.01264	.01216	.01170	.01125
3.5 +	.01081	.01038	.00997	.00957	.00917	.00879	.00843	.00807	.00772	.00739
4.0 +	.00706									

SIGMA=2.8 T=1.00

RADIAL PROFILE OF IEFF(RHO) AT EVENLY-SPACED VALUES OF RHO

IEFF(RHO + .05M)

RHO		M=0	M=1	M=2	M=3	M=4	M=5	M=6	M=7	M=8	M=9
0.0	+	.03965	.03964	.03961	.03956	.03949	.03940	.03929	.03916	.03903	.03883
.5	+	.03854	.03843	.03820	.03796	.03769	.03741	.03711	.03680	.03646	.03612
1.0	+	.03575	.03538	.03499	.03458	.03416	.03373	.03329	.03284	.03239	.03190
1.5	+	.03142	.03093	.03043	.02993	.02941	.02890	.02837	.02784	.02731	.02677
2.0	+	.02623	.02569	.02515	.02461	.02406	.02352	.02298	.02243	.02189	.02136
2.5	+	.02062	.02029	.01976	.01924	.01872	.01820	.01770	.01719	.01670	.01621
3.0	+	.01572	.01524	.01477	.01431	.01386	.01341	.01297	.01254	.01212	.01171
3.5	+	.01131	.01091	.01053	.01015	.00978	.00942	.00907	.00873	.00840	.00807
4.0	+	.00776									

SIGMA=3.0 T=1.00